Optimal Maintenance Policy and Fund Allocation in Water Distribution Networks

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Abstract: In maintenance of a water distribution network, the allocation of limited funds for maintenance among a network's pipes and the decision to repair or replace a single pipe when failure occurs are the most important issues. Although some researchers in the past have addressed these issues, more work is needed to support water authorities in making maintenance decisions. In this research paper, a mathematical model is developed that aims to support the decision to allocate funds among pipes of the network as well as the decision to repair or replace the pipes in the state of failure. The objective function of the model is to maximize the total weighted long-run availability of the whole system. The concept of hydraulic reliability is employed to determine the weight of pipes in the maintenance program. The deterioration behavior of the pipe is depicted by a semi-Markov process, and the Dantzig–Wolfe decomposition algorithm is applied to deal with the large-scale characteristic of the resulting program.

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Introduction

In maintenance management of water distribution networks, the total available funds are usually not enough to completely satisfy the upgrading requirement of all components of the network. A network's components therefore have to compete for the limited available funds.

Although there are some previous research works that guide the prioritization of maintenance actions in relation to several pipes within a water distribution network (Arulraj and Rao 1995; Quimpo and Shamsi 1991; Quimpo and Wu 1997), it is not simple to find an optimal maintenance policy as well as fund an allocation program for the whole network. This is due to the complexity in solving large-scale nonlinear programming problems that usually result from the determination of such an optimal policy.

Much research has been conducted in the past to help determine the optimal maintenance policy for water distribution networks. Shamir and Howard (1979) proposed the use of the regression technique based on historical records of main breaks to forecast the number of breaks in the future if the pipes were not replaced. Cost analysis is then conducted to determine the average time in which a pipe should be replaced so as to minimize the cost of maintenance. The cost-based approach is also employed in Kleiner et al. (1998) to construct a long-term rehabilitation strategy for water distribution networks. The cost-based approach is appropriate for pipes at the tertiary or secondary levels of the network (small pipes), where the break rate can be estimated by the use of a regression model and where pipe breaks do not result in major supply deficiency. For larger pipes at the primary level of some networks, the use of regression models to predict break rates might not be appropriate due to the existence of two stages of deterioration where the break rate increases very quickly when the pipes start to break.

Beside the maintenance cost, a network's availability/reliability is also an important criterion when establishing maintenance policy in water distribution networks. Depending on the network under consideration, water authorities might select either maintenance cost or availability/reliability as their objective in the maintenance policy. However, for primary levels of water distribution networks, which include larger pipes with very high supply capacity, pipe breaks might cause severe supply deficiency and the use of availability/reliability seems more appropriate. Li and Haimes (1992a,b) have considered this objective with their proposed model, where the deterioration behavior of a pipe is represented by the use of a semi-Markov process proposed by Andreou and Mark (1987). Li and Haimes (1992b) proposed the use of system availability, which is defined as the weighted sum of nodal availabilities, as an objective function to find the optimal repair/replacement policy based on a set of predefined maintenance actions at all states of a deteriorating network. In their work, nodal availability is defined as the availability from the source node to the demand node; in other words, it is the probability that the demand node is connected to the source node. Following this definition, the resulting model is a very sophisticated large-scale nonlinear program. Although a decomposition scheme has been proposed and successfully employed to solve the resulting nonlinear program, the model proposed by Li and Haimes (1992b) is difficult to apply in actual water distribution networks due to the complex relationship between nodal availability and availability

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of a network's pipes, especially when the network is rather large. Another impractical aspect of the model proposed by Li and Haimes (1992a,b) is that in the optimal maintenance schedule resulting from their approach, the maintenance action taken when the pipe is at some failure state is not a deterministic one—it is randomly selected among a predefined set of actions with the selection probability for each action determined from a discrete distribution.

In this research, the model proposed by Luong and Nagarur (2001) to find an optimal replacement policy under limited funds for a single pipe in a water distribution network will be employed so that it can help find an optimal fund allocation and replacement policy for all pipes in a water distribution network. It is assumed that any pipe in the network can be considered a link connecting two nodes in the network. It is also assumed that failure behaviors of pipes in the network are independent—that is, the failure of one specific pipe is not the result of failures of other pipes in the water distribution network. As shown later, the nonlinear program developed in this research can be converted to a linear program. A traditional decomposition algorithm, the Dantzig–Wolfe decomposition algorithm, can then be applied to deal with the large-scale characteristic of the resulting linear program.

Model for Single Pipe

This paper is an extension of the methodology proposed by Luong and Nagarur (2001) to deal with maintenance problem of the whole network. In the research of Luong and Nagarur (2001), the deteriorating behavior of a pipe has been modeled as a semi-Markov process in which the state space represents the states of the pipe as new, operating with n repairs undergone—under repair or under replacement. It is assumed in the above research that the pipe will be replaced according to the following criteria:

1. The pipe will be replaced if it has experienced more than N breaks regardless of its age; and
2. If the pipe is experiencing a break, it will be replaced if its operation time in the previous operating state has reached a specified threshold duration for replacement. Otherwise, the pipe will be repaired.

The states of a semi-Markov process representing the behavior of a pipe over time are defined as follows:

- State 0: New pipe is operating either after installation or replacement.
- State $2n$ $(n=1,2,\ldots,N)$: The pipe is in operating state after undergoing n repairs.
- State $2n-1$ $(n=1,2,\ldots,N)$: The pipe is in repaired state the nth time. This state is reached when the pipe experiences a break and its operation time in the preceding operating state (i.e., state $2n-2$), is smaller than a specified threshold duration for replacement.
- State $2N+1$: The pipe is in replacement state. This may be the replacement when the pipe experiences a break and its operation time in the preceding operating state exceeds the specified threshold duration for replacement, or when the pipe experiences the $(N+1)$st break.

If T_{2n-2} $(n=1,2\ldots,N)$ represents the threshold duration for replacement of operating state $2n-2$ $(n=1,2,\ldots,N)$, beyond which the pipe will be replaced at the next failure, then the maintenance policies, which are considered as the combinations of $[N,T_{2n-2}\ (n=1,2,\ldots,N)]$, can be established and investigated.

The selection of N, the number of pipe breaks after which the pipe will be replaced, is one of the most important tasks in determining the maintenance policy. In the research of Luong and Nagarur (2001), sensitivity analysis was conducted on N to investigate the effect of N, and guidance for the selection of N, was also given. In general, the finding is that, at a fixed value of allowable cost rate, there may exist more than one value of N that gives the same optimal result in terms of availability. However, the optimal maintenance policy with a smaller value of N is preferred due to the simple maintenance scheme resulting from it.

Details on the determination of transition probabilities of the semi-Markov process; residence times of the process in operating states, repaired states, and replacement state; and limiting probabilities can be referred to in Luong and Nagarur (2001).

It is also noted that only direct costs of maintenance are considered in the research of Luong and Nagarur (2001), and the total cost rate for the single pipe problem can be determined by

$$q=\sum_{n=0}^{N} q_{2n-1}\phi_{2n-1}+\frac{c_{sr}}{t_{sr}}\phi_{2N+1}=\frac{c_r}{t_r}\sum_{n=0}^{N}\phi_{2n-1}+\frac{c_{sr}}{t_{sr}}\phi_{2N+1} \qquad (1)$$

where q_{2n-1}=cost rate associated with repaired state $2n-1$ $(n=1,2,\ldots,N)$; c_r and c_{sr}=average repair and replacement costs of the pipe under consideration; t_r and t_{sr}=average repair and replacement times; and ϕ_{2n-1} and ϕ_{2N+1}=limiting probabilities associated with the repaired state $2n-1$ and the replacement state $2N+1$.

Model for the Whole Network

Objective Function

Similar to the work of Li and Haimes (1992b), the concept of system availability will be used as the objective function in this research. However, it is different from the work of Li and Haimes (1992b) in that the system reliability used here is defined as the total weighted long-run availability of all pipes in the network. The optimal policy will then be determined so that the total weighted long-run availability of all pipes in the network is maximized. The objective function is, hence, expressed as

$$\text{Maximize} \quad Z=\sum_{k=1}^{K} w^k\phi^k=\sum_{k=1}^{K} w^k\sum_{n=1}^{N^k}\phi_{2n}^k \qquad (2)$$

where ϕ^k=long-run availability of pipe k; ϕ_{2n}^k=long-run availability in operating state $2n$ of pipe k; and w^k=weight of pipe k in the objective function. In general, values of w^k's may be set beforehand by the water authority such that the pipes that are more important in the network's structure will receive larger weights. In comparison with the definition of system availability used by Li and Haimes (1992b), which is based mainly on the structural connectivity between each demand node and the source node, the concept of system availability used here allows the incorporation of hydraulic considerations in selecting the best policy through an appropriate definition of weight. Furthermore, the system reliability defined as such can be easily derived for real water distribution networks with a large number of pipes.

In order to determine the weights, the model proposed in this research will use the concept of *hydraulic system reliability* as an indicator for the important level of a pipe in the network. It is noted that if the system reliability is reduced largely when a specific pipe, say pipe k, is in the down state (repaired state or replacement state), then the weight assigned to that pipe, or w^k, should be larger in comparison with other pipes with smaller reductions of system reliability in the down states. If R^k is the system reliability when pipe k is in the down state from the above discussion, it can be seen that the supplement of system reliability when pipe k is in the down state, or $1-R^k$, can be used as the weight of that pipe in calculation of total availability. In the normalized form, the weight of pipe k can be expressed as

$$w^k = \frac{1-R^k}{\sum_{k=1}^{K}(1-R^k)} \text{ so that } \sum_{k=1}^{K} w^k = 1 \quad (3)$$

Hydraulic Reliability

In the past, various concepts of reliability and approaches for reliability assessment have been proposed and used for analyzing the reliability of water distribution networks. Based on the topology of the network, Goulter and Coals (1986) introduced the probability of simultaneous failure of all links connected to a node, which is termed as the probability of *node isolation*. Wagner et al. (1988) introduced *reachability* as the probability that a given demand node is connected to at least one source node, and *connectivity* as the probability that all demand nodes are connected to at least one source node.

Taking into consideration the hydraulic requirements, the basic concept of system reliability commonly perceived is that the water should be provided from sources to each demand point at the desired time, at the desired pressure, and at the desired flow rate. The concept of hydraulic reliability has been widely used in determining system reliability (Hobbs and Beim 1988; Duan and Mays 1990; Fujiwara and De Silva 1990; Fujiwara and Tung 1991; Fujiwara and Ganesharajah 1993; Gupta and Bhave 1994, 1996).

In the context of this research, the definition of Fujiwara and De Silva (1990) on hydraulic reliability is employed. According to this definition, reliability is expressed as the complement of the ratio of the minimum total shortfall in supply to the total required demand. In other words, it is the ratio of the maximum supply flow rate to the total required demand.

At the node level, the hydraulic nodal reliability of a demand node m in the distribution network can be expressed as

$$R_m = \frac{q_m}{q_m^r} \quad (4)$$

where q_m and q_m^r=outflow and the required flow rate at demand node m.

The corresponding hydraulic system reliability can then be expressed as

$$R = \frac{\sum_{m\in D} q_m}{\sum_{m\in D}^{M} q_m^r} = \frac{\sum_{m\in S} Q_m}{\sum_{m\in D}^{M} q_m^r} \quad (5)$$

where S and D=set of source nodes and the set of demand nodes, respectively; and Q_m=supply flow rate from source node m.

The relationship between the nodal reliability and system reliability defined above can be expressed as

$$R = \frac{\sum_{m\in D} R_m q_m^r}{\sum_{m\in D}^{M} q_m^r} \quad (6)$$

It is noted that the system reliability defined in Eq. (6) is the weighted average of all nodal reliabilities, in which the weights are the required flow rates at demand nodes. In practice, other weights can be assigned to demand nodes depending on water authority and on the important level of each node. It can be seen that the system reliability defined in Eq. (6) focuses on the ability of the system to supply water as much as possible to the demand points. This can lead to the fact that some nodes in the network will receive enough flow rates while other nodes may receive a little amount of their demand or even not receive any water at all in deficient conditions. This situation can happen in water distribution networks with spanning tree structures. In such a network, it may be better to express system reliability as the geometric mean of nodal reliabilities (Bao and Mays 1990). The relationship between system reliability and nodal reliabilities in that case, hence, can be expressed as

$$R = \sqrt[M_D]{\Pi_{m\in D} R_m} \quad (7)$$

where M_D=number of demand nodes in the distribution network.

In the context of this research work, the definition of system reliability in Eq. (6) will be taken into consideration due to the fact that most of the water distribution networks have looped structures. Nodal reliabilities and system reliability will then be evaluated. The *maximum flow model* (Fujiwara and De Silva 1990) incorporated with the concept of *pressure dependent demand* (Fujiwara and Ganesharajah 1993) will be employed in the evaluation of nodal reliabilities and system reliability.

Based on the network's structure resulting from the separation of pipe k from the network, the system reliability when pipe k is in down state, or R^k, can be evaluated. All R^k values for all pipes of the distribution network are then used to determine the weights in the model developed to find an optimal maintenance policy for the whole water distribution network.

Model Formulation

The following symbols are used in this section:

k=index of pipe in the network $(k=1,2,\ldots,K)$;

N^k=predefined number for pipe k so that if that pipe experiences more than N^k breaks, it will be replaced;

q_i^k=cost rate of pipe k in state i $(k=1,2,\ldots,K;\ i=0,1,\ldots,2N^k+1)$;

q_a=allowable cost rate for the whole distribution network;

T_{2n}^k=threshold duration for replacement of operating state $2n$ of pipe k $(k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1)$;

P_{ji}^k=transition probability from state j to state i of pipe k $(k=1,2,\ldots,K;\ i,j=0,1,\ldots,2N^k+1)$;

ϕ_n^k=steady-state probability of pipe k in state n $(k=1,2,\ldots,K;\ n=0,1,\ldots,2N^k+1)$;

ϕ^k=long-run availability of pipe $k\,(k=1,2,\ldots,K)$; $\phi^k=\sum_{n=0}^{N^k}\phi_{2n}^k$; and

τ_n^k=expected residence time of pipe k in state n $(k=1,2,\ldots,K;\ n=0,1,\ldots,2N^k+1)$.

Based on the maintenance model developed for a single pipe proposed by Luong and Nagarur (2001), the mathematical model, which aims to determine the optimal fund allocation and maintenance policy for the whole distribution network, can be formulated as follows

$$(\mathbf{Q})\quad \text{Maximize}\quad Z=\sum_{k=1}^{K} w^k \phi^k = \sum_{k=1}^{K} w^k \sum_{n=0}^{N^k} \phi_{2n}^k \tag{8a}$$

subject to

$$\sum_{j=0}^{2N^k+1} P_{ji}^k \frac{\phi_j^k}{\tau_j^k} = \frac{\phi_i^k}{\tau_i^k} \quad k=1,2,\ldots,K;\ i=0,1,2,\ldots,2N^k+1 \tag{8b}$$

$$\sum_{i=0}^{2N^k+1} \phi_i^k = 1 \quad k=1,2,\ldots,K \tag{8c}$$

$$\sum_{k=1}^{K} \sum_{i=0}^{2N^k+1} q_i^k \phi_i^k \leqslant q_a \tag{8d}$$

$$\begin{aligned} &T_{2n}^k \geqslant 0 \quad k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1 \\ &\phi_i^k \geqslant 0 \quad k=1,2,\ldots,K;\ i=0,1,2,\ldots,2N^k+1 \end{aligned} \tag{8e}$$

It is noted that the above program can be converted to a linear program by introducing the following set of variables:

$$\varphi_{2n,2n+1}^k = P_{2n,2n+1}^k \phi_{2n}^k \tag{9a}$$

$$\varphi_{2n,2N^k+1}^k = P_{2n,2N^k+1}^k \phi_{2n}^k \tag{9b}$$

where $k=1,2,\ldots,K$; and $n=0,1,\ldots,N^k-1$ (see Luong and Nagarur 2001).

From the fact that transition probabilities of the embedded Markov chains used to represent the deterioration behavior of a network's pipes are functions of $T_{2n}^k(k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1)$ (see Luong and Nagarur 2001), the limiting probabilities determined from Eqs. (8*b*) and (8*c*), are also functions of $T_{2n}^k(k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1)$ and hence, the optimization program (Q) depends only on $T_{2n}^k(k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1)$. However, it should be noted that the optimization program (Q) can be solved with respect to $\phi_i^k(k=1,2,\ldots,K;\ i=0,1,\ldots,2N^k+1)$; $\varphi_{2n,2n+1}^k$ and $\varphi_{2n,2N^k+1}^k(k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1)$. When the solution of the optimization program is found, all transition probabilities of the embedded Markov chains can be determined from Eqs. (9*a*) and (9*b*) and hence, the decision variable $T_{2n}^k(k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1)$ can be uniquely determined from the corresponding transition probabilities.

In the resulting linear program, another set of constraints is also included in the original formulation (Luong and Nagarur 2001)

$$\varphi_{2n,2n+1}^k + \varphi_{2n,2N^k+1}^k = \phi_{2n}^k \quad k=1,2,\ldots,K;\ n=0,1,\ldots,N^k-1 \tag{10}$$

With the new set of variables introduced above, it is noted that program (Q) is a *block angular system* with only one *linking* constraint [Eq. (8*d*)]. This program can, hence, be solved by applying a decomposition technique so that the original program can be separated into K subprograms corresponding to K pipes of the water distribution network in each step of the decomposition scheme. The decomposition algorithm proposed by Dantzig and Wolfe (1960) will be employed for the solution process.

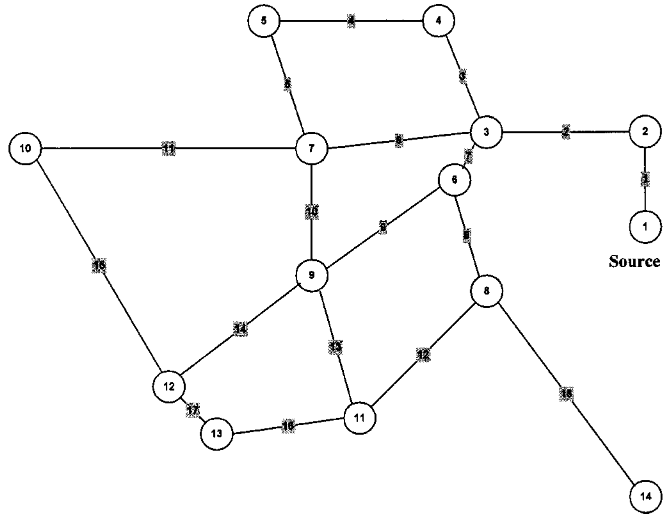

Fig. 1. Primary level of the water distribution network of Ho Chi Minh City, Vietnam

Numerical Experiments

In order to illustrate the applicability of the proposed model, the optimal fund allocation and maintenance policy for the primary level of the water distribution network of Ho Chi Minh City in Vietnam is investigated. The network under investigation has only 1 source node, 13 demand nodes, and 18 links, as shown in Fig. 1.

Characteristics of the network's links (i.e., link lengths, link diameters, and roughness coefficients) are given in Table 1. The required flow rates, minimum heads, and required heads at various demand nodes are given in Table 2. The elevation of source

Table 1. Link Information

Link	From node	To node	Length (m)	Roughness coefficient	Diameter (m)
1	1	2	3207.62	120	2.00
2	2	3	5700.56	120	2.00
3	3	4	3706.05	120	0.60
4	4	5	5232.06	120	0.40
5	5	7	4076.13	120	0.50
6	3	7	5038.29	120	0.80
7	3	6	2848.51	120	2.00
8	6	8	3462.08	120	1.50
9	6	9	4399.45	120	1.20
10	7	9	4130.38	120	0.20
11	7	10	9350.94	120	0.60
12	8	11	4251.16	120	1.20
13	9	11	4400.88	120	0.20
14	9	12	5694.35	120	0.80
15	10	12	8810.22	120	0.40
16	11	13	5254.22	120	0.90
17	12	13	1857.85	120	0.20
18	8	14	10089.30	120	0.90

Table 2. Node Information

Node	Require flow rate (m^3/hr)	Minimum head (m)	Required head (m)
1	—	—	—
2	354.14	26.0	40.0
3	4307.66	12.5	26.5
4	586.92	16.0	30.0
5	1173.84	12.0	26.0
6	3677.68	13.0	27.0
7	1731.60	16.0	30.0
8	1567.80	12.5	26.5
9	3493.81	15.0	29.0
10	1117.72	11.0	25.0
11	3064.74	13.0	27.0
12	2424.24	13.0	27.0
13	2341.54	12.5	26.5
14	1751.12	12.5	26.5

node, node 1, is 35*m* and the maximum head raised from the pumping system is 80*m*.

The average repair time for each repair action is estimated from the regression formula:

$$\text{Repair time(hrs)} = 4.9835^{*}(\text{Diameter(cm)})^{0.4276}$$

The replacement time of a link depends not only on the length of the link but also on its diameter, and is estimated from the following formula [Water Works Authority (1998)]:

$$\text{Replacement time(hrs)} = 65.1048^{*}\text{Length(km)}^{*}(\text{Diameter(cm)})^{0.1789}$$

The system availability with respect to various structures of the network when one link is removed from the network is investigated and the results are shown in Table 3. From these results, it can be seen that Link 10, Link 13, and Link 17 are not important in the network because when one among these links is separated

Table 3. System Reliability with Respect to Missing Links and Weights of Various Links of the Network

Missing link	System reliability	Weight
1	0.00000000	0.253615
2	0.01283450	0.250360
3	0.97321336	0.006794
4	0.99309846	0.001750
5	0.97974666	0.005137
6	0.88314138	0.029637
7	0.33404516	0.168896
8	0.65088119	0.088542
9	0.76605120	0.059333
10	1.00000000	—
11	0.97344261	0.006735
12	0.77994169	0.055810
13	1.00000000	—
14	0.88923018	0.028093
15	0.99747612	0.000640
16	0.88737771	0.028563
17	1.00000000	—
18	0.93653709	0.016095

from the network's structure, the supply capacity from the source node to every demand node is still ensured. The existence of these links in the design of the network is to provide a specific level of topological redundancy in the network at the initial design phase. The most important links in the network are Links 1, 2, and 7. When one among these links is separated from the network, the system reliability reduces to a very low level, at which many demand nodes cannot receive their required flow rates or even not receive any water at all.

From the above analysis, it can be seen that Links 10, 13, and 17 can be ignored. These links do not make any contribution to the total weighted system availability, which is the objective function of the proposed model, and hence, should be discarded from further consideration. A maintenance policy corresponding to the minimum required cost rate as investigated in the paper of Luong and Nagarur (2001) or other maintenance policies may be applied to these links. The weights of the remaining links in the maintenance program are then calculated and presented in Table 3.

Due to the fact that all pipes in the primary network of Ho Chi Minh City are of the same material type and the environmental condition at Ho Chi Minh City does not change much for the whole year (in terms of temperature, humidity, etc.), the deterioration of the pipes is mainly due to the aging process. In this research, it is assumed that the deterioration processes are similar for all links of the network. It is also employed that residence times of pipes (in years) in operating states 0, 2, and 4 follow Weibull distributions with a common shape parameter of 4 and the corresponding scale parameters of 0.012754, 0.015686, and 0.015843, respectively. For other operating states, the Poisson-type model with rate 0.5 is used. Actually, different deteriorating patterns exist for different pipes of the network. However, due to lack of information on break records, the above parameters, which are estimated only for Link 7 of the network, are assumed to hold true for other links (Water Works Authority 1998). This is a limitation of the paper.

Although the assumption on similar deterioration behaviors for all pipes of the water distribution network as mentioned above is not realistic and hence, the resulting optimal policy may not be applicable, the analyses in this section still give some valuable insights for water authorities of Ho Chi Minh City in making fund allocation and maintenance decisions. These analyses show how different priorities will be given to various pipes of the network based on their locations, or their relative importance level in the network's structure; their required times for repair and replacement; and their costs of maintenance (i.e., repair cost and replacement cost).

The optimal fund allocation and maintenance policy for the whole network will be first investigated with the value of $N=4$ in this research work, which is currently applied in the water distribution network of Ho Chi Minh City. In the former research of Luong and Nagarur (2001) for a single pipe (Link 7 of the network), it has been found that, at a fixed value of allowable cost rate, there may exist more than one value of N that gives the same optimal result in terms of availability, or equivalently, there may exist more than one optimal maintenance policy at a given allowable cost rate. However, the optimal maintenance policy with a smaller value of N is preferred due to the simple maintenance scheme resulting from it. It is also found that it is not always economical to replace a pipe when it enters the late stage of deterioration the first time (i.e., when the pipe experiences the fourth break). From these results, we can see that the practice of using $N=4$ in the network of Ho Chi Minh City is a reasonable one, particularly for Link 7 of the network. The use of $N=4$ for

Table 4. Minimum Required Cost Rate, Maximum Cost Rate, and the Corresponding Optimal Availability

	Minimum		Maximum	
Link	Cost rate ($1000/year)	Optimal availability	Cost rate ($1000/year)	Optimal availability
1	23.38	0.99956120	23.86	0.99961064
2	41.50	0.99930996	42.37	0.99935398
3	3.85	0.99962973	3.92	0.99965696
4	3.29	0.99954642	3.35	0.99956626
5	3.39	0.99961544	3.45	0.99963973
6	7.77	0.99949080	7.92	0.99951965
7	20.77	0.99959740	21.19	0.99964763
8	15.51	0.99956640	15.82	0.99960914
9	13.51	0.99950240	13.78	0.99953903
11	9.66	0.99917110	9.86	0.99918845
12	13.05	0.99951604	13.31	0.99955296
14	8.78	0.99943468	8.95	0.99946232
15	5.53	0.99927604	5.64	0.99929006
16	9.90	0.99945875	10.10	0.99948895
18	18.97	0.99903650	19.36	0.99905760

other pipes of the network might be questioned. However, due to the fact that all pipes in the primary network of Ho Chi Minh City are of the same material type, they are installed at the same time on a flat terrain and worked under the same environmental conditions; therefore the use of $N=4$ for all pipes in the network is not an unreasonable choice.

The minimum required and maximum cost rates for each pipe and the optimal availabilities associated with these cost rates are then investigated and the results are shown in Table 4. The optimal policy when the total allowable cost rate is set at 199.5 ($1000/year) is investigated next, and the optimal allocated funds, as well as the availability of each link, are shown in Table 5.

From Table 5 it can be seen that only Link 1 is given the maximum funds. Link 7 is given an intermediate fund level between its minimum required cost rate and maximum cost rate. All other links, including Link 2, are given the minimum funds. At first glance, looking at the competition between Link 2 and Link 7 for funding, this result seems to be contrary to common sense—in other words, Link 7 should be given the minimum funds while Link 2 should be given the intermediate fund level in the above case. The fact that Link 7 was preferred over Link 2 can be explained from the higher cost associated with Link 2. This cost is nearly double the associated cost of Link 7 and hence, although Link 2 has a higher weight, this weight is not sufficient to offset the difference in cost. In more detail, this contrary result can also be explained as follows: although Link 2 is structurally more important than Link 7 and is assigned a larger weight in the objective function, it can be seen from Table 4 that Link 2 always requires a much higher cost rate for maintenance than Link 7, while its corresponding availability is smaller than the availability of Link 7. Therefore, Link 7 is preferable in the competition for allocated maintenance funds in the optimal maintenance policy. Furthermore, it is also noted that the long-run proportion of time that both Link 2 and Link 7 are in operating states in the above optimal maintenance policy can be approximately expressed by the product of the two links' availabilities—that is, $\phi^{2*}\phi^{7}=0.99892502$. If it is now assumed that Link 7 is given the minimum funds and the amount of funds reduced from Link 7 is assigned to Link 2, then the corresponding optimal availabilities of these two links are $\hat{\phi}^{2}=0.99930996$, $\hat{\phi}^{7}=0.99959740$ and $\hat{\phi}^{2*}\hat{\phi}^{7}=0.99891502<\phi^{2*}\phi^{7}$. Hence, the optimal maintenance policy presented in Table 5 is better in the sense that the long-run proportion of time that both Link 2 and Link 7 are in operating states is higher, or equivalently, the long-run proportion of time that all links of the network are in operating states, which is the product of all the links' availabilities, is higher. The higher priority given to a specific pipe in the optimal maintenance policy is the result of a combined consideration that depends not only on the structural importance level of the pipe but also on the contribution of that pipe to the availability of the network as a whole in conjunction with its required cost rate.

The corresponding threshold durations for replacement of various links in conjunction with the optimal maintenance policy are also investigated and the results are as follows: The link that is assigned the funds equal to its maximum cost rate (i.e., Link 1), will be replaced when it experiences the third break; when it experiences the first or second break, it will be repaired. The links with the minimum required cost rates assigned (i.e., all other links except Link 7), will always be repaired at failure and be replaced only if the fifth break occurs. Link 7 will be replaced when its operating time after the third break exceeds 64 years or if the fifth break occurs. From the operational standpoint, the above results determine a clear maintenance plan for each pipe. However, it should be noted that the most important decision in the network problem considered in this paper is the allocation of maintenance funds for each individual pipe. The values of T_{2n}^{k} are not the main target of this research. This comes from the finding of the former work for a single pipe that at a fixed value of allowable cost rates, there may exist more than one value of N that gives the same optimal result for a single pipe. Therefore, although the problem has been solved with $N=4$ for all pipes, a different value of N can be used for a specific pipe without changing the optimal availability of the whole network that has been found. All possible values of N and the resulting values of the corresponding T_{2n}^{k} can be determined by running the model of single pipe proposed by Luong and Nagarur (2001).

Table 5. Optimal Allocated Funds and Availabilities of Various Links at Total Cost Rate $q_a=199.5$ ($1000/year)

Link	Allocated funds ($1000)	Availability
1	23.86	0.99961064
2	41.50	0.99930996
3	3.85	0.99962973
4	3.29	0.99954642
5	3.39	0.99961544
6	7.77	0.99949080
7	20.92	0.99961479
8	15.51	0.99956640
9	13.51	0.99950240
11	9.66	0.99917110
12	13.05	0.99951604
14	8.78	0.99943468
15	5.53	0.99927604
16	9.90	0.99945875
18	18.97	0.99903650
Total weighted system availability:		**0.99949522**

Table 6. Summary on Optimal Allocated Funds of Various Links at Various Levels of Total Allowable Cost Rate

	Total allowable cost rate ($1000/year)								
Link	199.1	199.5	199.9	200.3	200.7	201.1	201.5	201.9	202.3
1	23.61	23.86	23.86	23.86	23.86	23.86	23.86	23.86	23.86
2	41.50	41.50	41.63	42.03	42.37	42.37	42.37	42.37	42.37
3	3.85	3.85	3.85	3.85	3.85	3.85	3.85	3.85	3.92
4	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29	3.29
5	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.39	3.45
6	7.77	7.77	7.77	7.77	7.77	7.77	7.79	7.92	7.92
7	20.77	20.92	21.19	21.19	21.19	21.19	21.19	21.19	21.19
8	15.51	15.51	15.51	15.51	15.56	15.82	15.82	15.82	15.82
9	13.51	13.51	13.51	13.51	13.51	13.66	13.78	13.78	13.78
11	9.66	9.66	9.66	9.66	9.66	9.66	9.66	9.66	9.66
12	13.05	13.05	13.05	13.05	13.05	13.06	13.31	13.31	13.31
14	8.78	8.78	8.78	8.78	8.78	8.78	8.78	8.95	8.95
15	5.53	5.53	5.53	5.53	5.53	5.53	5.53	5.53	5.53
16	9.90	9.90	9.90	9.90	9.90	9.90	9.90	10.00	10.10
18	18.97	18.97	18.97	18.97	18.97	18.97	18.97	18.97	19.15

Sensitivity analysis with respect to various levels of total available cost rates is then investigated, and summaries on optimal allocated funds and availabilities of various links at various levels of total allowable cost rates are shown in Tables 6 and 7.

From the results, it can be seen that important links such as Links 1, 2, 7, 8, 9, and to some extent, Link 12 are always given higher priorities in comparison with other links of the network. The structural importance levels of these links are so high that their large required cost rates are inconsequential in the competition between them and other links for maintenance funds. However, the competition between the links with lesser structural importance levels is rather complicated, as can be seen in the cases q_a=201.5, 201.9, and 202.3 for Links 3, 5, 6, 14, 16, and 18. The competition between these links does not only depend on their structural importance levels but on their required maintenance cost rates and the corresponding contribution of their availabilities to the system availability as well. The remaining links in the network—Links 4, 11, and 15—are the least important links. This can be seen from the smallest weights of these links in the objective function, which represent their structural importance levels. Furthermore, it can also be seen from Table 4 that the contributions of Links 11 and 15 to the system availability are rather small in comparison with other links. Links 4, 11, and 15 are, therefore, given the least priorities in the optimal maintenance policy.

It is also noted from Table 7 that the system availability increases when total allowable cost rate increases. However, the marginal system availability, which is the increase of system availability when total allowable cost rate increase by one unit, decreases. In other words, the system availability is an increasing concave function of total allowable cost rate.

Table 7. Summary on Optimal Availabilities of Various Links at Various Levels of Total Allowable Cost Rate

	Total allowable cost rate ($1000/year)								
Link	199.1	199.5	199.9	200.3	200.7	201.1	201.5	201.9	202.3
1	0.99958427	0.99961064	0.99961064	0.99961064	0.99961064	0.99961064	0.99961064	0.99961064	0.99961064
2	0.99930996	0.99930996	0.99931630	0.99933661	0.99935398	0.99935398	0.99935398	0.99935398	0.99935398
3	0.99962973	0.99962973	0.99962973	0.99962973	0.99962973	0.99962973	0.99963229	0.99963012	0.99965681
4	0.99954642	0.99954642	0.99954642	0.99954642	0.99954642	0.99954642	0.99954642	0.99954642	0.99954642
5	0.99961544	0.99961544	0.99961544	0.99961544	0.99961544	0.99961544	0.99961544	0.99961697	0.99963971
6	0.99949080	0.99949080	0.99949080	0.99949080	0.99949080	0.99949080	0.99949371	0.99951965	0.99951965
7	0.99959740	0.99961479	0.99964763	0.99964763	0.99964763	0.99964763	0.99964763	0.99964763	0.99964763
8	0.99956640	0.99956640	0.99956640	0.99956640	0.99957438	0.99960914	0.99960914	0.99960914	0.99960914
9	0.99950240	0.99950240	0.99950240	0.99950240	0.99950240	0.99952253	0.99953903	0.99953903	0.99953903
11	0.99917110	0.99917110	0.99917110	0.99917110	0.99917110	0.99917110	0.99917110	0.99917110	0.99917110
12	0.99951604	0.99951604	0.99951604	0.99951604	0.99951604	0.99951613	0.99955283	0.99955296	0.99955296
14	0.99943468	0.99943468	0.99943468	0.99943468	0.99943468	0.99943468	0.99943468	0.99946232	0.99946232
15	0.99927604	0.99927604	0.99927604	0.99927604	0.99927604	0.99927604	0.99927604	0.99927604	0.99927604
16	0.99945875	0.99945875	0.99945875	0.99945875	0.99945875	0.99945875	0.99945875	0.99947369	0.99948895
18	0.99903650	0.99903650	0.99903650	0.99903650	0.99903650	0.99903650	0.99903650	0.99903650	0.99904617
Total weighted system availability	**0.99948559**	**0.99949522**	**0.99950235**	**0.99950744**	**0.99951249**	**0.99951677**	**0.99951990**	**0.99952187**	**0.99952276**

Conclusions

In this research, a mathematical model is developed to find the optimal fund allocation and maintenance policy for the water distribution network as a whole. A new concept of total weighted system availability is proposed in which hydraulic reliabilities of the distribution network in various deficient structures are used as the weights in calculation of system availability.

The proposed model is not too complicated and it can be converted to a linear program with a specific structure such that a decomposition technique can be easily applied in dealing with large networks. The allocation of limited funds to different pipes of the network is found to depend on three main factors: (1) the importance levels of the pipes in the network's structure; (2) the maintenance cost rate required by the pipes in comparison with each other; and (3) the individual optimal availability of each pipe, which represents the contribution of the pipe to the total system availability, in relationship to the provided maintenance funds.

It should be noted that the reliability of the results presented in our numerical experiments is not high. This comes from the fact that historical records of only Link 7 have been used to develop regression equations to estimate repair time, replacement time, and also scale parameters of the Weibull distribution, but due to lack of information on other links, these input parameters have been used for the whole network. Due to this limitation, the numerical experiments are only an illustration for the applicability of the proposed methodology in dealing with maintenance of water distribution networks.

Actually, different failure patterns in various pipes will also have an effect on the optimal maintenance policy. This aspect is not considered in detail in this research work due to lack of information. It is also noted that the time value of money has not been considered in the model developed here.

The application of our paper might be limited due to the requirement of a vast amount of accurate input data, which are usually difficult to acquire in practice. This is also another limitation that might affect the applicability of the proposed methodology. However, the model presented in this paper can serve as a guide to water authorities in making replacement decision of network's pipes.

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