Optimal Operation of Multireservoir Systems: State-of-the-Art Review

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Abstract: With construction of new large-scale water storage projects on the wane in the U.S. and other developed countries, attention must focus on improving the operational effectiveness and efficiency of existing reservoir systems for maximizing the beneficial uses of these projects. Optimal coordination of the many facets of reservoir systems requires the assistance of computer modeling tools to provide information for rational management and operational decisions. The purpose of this review is to assess the state-of-the-art in optimization of reservoir system management and operations and consider future directions for additional research and application. Optimization methods designed to prevail over the high-dimensional, dynamic, nonlinear, and stochastic characteristics of reservoir systems are scrutinized, as well as extensions into multiobjective optimization. Application of heuristic programming methods using evolutionary and genetic algorithms are described, along with application of neural networks and fuzzy rule-based systems for inferring reservoir system operating rules.

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Introduction

According to the World Commission on Dams (WCD 2000), many large storage projects worldwide are failing to produce the level of benefits that provided the economic justification for their development. This may be due in some instances to an inordinate focus on project design and construction, with inadequate consideration of the more mundane operations and maintenance issues once the project is completed. Performance related to original project purposes may also be undermined when new unplanned uses arise that were not originally considered in the project authorization and development. These might include municipal/industrial water supply, minimum streamflow requirements for environmental and ecological concerns, recreational enhancement, and accommodating shoreline encroachment and development. Although there may exist some degree of commensurability among these diverse project purposes, there is more often conflict and competition, particularly during pervasive drought conditions. In addition, performance of publically owned reservoir systems is often restricted by complex legal agreements, contracts, federal regulations, interstate compacts, and pressures from various special interests.

With construction of new large-scale water storage projects at a virtual standstill in the U.S. and other developed countries, along with an increasing mobilization of opposition to large storage projects in developing countries, attention must focus on improving the operational effectiveness and efficiency of existing reservoir systems for maximizing the beneficial uses of these projects. In addition, many of the adverse impacts of large storage projects on aquatic ecosystems can be minimized through improved operations and added facilities, as demonstrated by the Tennessee Valley Authority (TVA) (Higgins and Brock 1999). Construction of bottom outlets or selective withdrawal structures can pass sediments downstream and improve water quality conditions. Unfortunately, many existing reservoir operational policies fail to consider a multifacility system in a fully integrated manner, but rather emphasize operations for individual projects. However, the need for integrated operational strategies confronts system managers with a difficult task. Expanding the scope of the working system for more integrated analysis greatly multiplies the potential number of alternative operational policies. This is further complicated by conflicting objectives and the uncertainties associated with future hydrologic conditions, including possible impacts of climate change.

Optimal coordination of the many facets of reservoir systems requires the assistance of computer modeling tools to provide information for rational operational decisions. Computer simulation models have been applied for several decades to reservoir system management and operations within many river basins. Many models are customized for the particular system, but there is also substantial usage of public domain, general-purpose models such as *HEC 5* (Hydrologic Engineering Center 1989), which is being updated as *HEC RESSIM* to include a Windows-based graphical user interface (Klipsch et al. 2002). Spreadsheets and generalized dynamic simulation models such as *STELLA* (High Performance Systems, Inc.) are also popular (Stein et al. 2001). Other similar system dynamics simulation models include *POWERSIM* (Powersim, Inc.) applied by Varvel and Lansey (2002), and *VENSIM* (Ventana Systems, Inc.) applied by Caballero et al. (2001). These simulation or *descriptive* models help answer *what if* questions regarding the performance of alternative operational strategies. They can accurately represent system operations and

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are useful for Monte Carlo analysis in examining long-term reliability of proposed operating strategies. They are ill-suited, however, to *prescribing* the best or optimum strategies when flexibility exists in coordinated system operations. Prescriptive optimization models offer an expanded capability to systematically select optimal solutions, or families of solutions, under agreed upon objectives and constraints.

The purpose of this paper is to assess the state-of-the-art in reservoir system optimization models and consider future directions. This is an update of a review that appeared in *Water Resources Update* published by The Universities Council on Water Resources (UCOWR) (Labadie 1997). The focus is primarily on optimization of systems of reservoirs, rather than a single reservoir. This is not meant to imply that single reservoir optimization is unimportant, but rather the substantial technological challenges and rewards abide with integrated optimization of interconnected reservoir systems. Optimization methods designed to prevail over the high-dimensional, dynamic, nonlinear, and stochastic characteristics of reservoir systems are scrutinized, as well as extensions into multiobjective optimization. Heuristic programming methods using evolutionary and genetic algorithms are described, along with the application of artificial neural networks and fuzzy rule-based systems for inferring reservoir system operating policies.

Overcoming Hindrances to Reservoir System Optimization

Despite several decades of intensive research on the application of optimization models to reservoir systems, authors such as Yeh (1985) and Wurbs (1993) have noted a continuing gap between theoretical developments and real-world implementations. Possible reasons for this disparity include: (1) many reservoir system operators are skeptical about models purporting to replace their judgment and prescribe solution strategies and feel more comfortable with use of existing simulation models; (2) computer hardware and software limitations in the past have required simplifications and approximations that operators are unwilling to accept; (3) optimization models are generally more mathematically complex than simulation models, and therefore more difficult to comprehend; (4) many optimization models are not conducive to incorporating risk and uncertainty; (5) the enormous range and varieties of optimization methods create confusion as to which to select for a particular application; (6) some optimization methods, such as dynamic programming, often require customized program development; and (7) many optimization methods can only produce optimal period-of-record solutions rather than more useful conditional operating rules. Optimal period-of-record solutions are criticized in the *Engineer Manual on Hydrologic Engineering Requirements for Reservoirs* (U.S. Army Corps of Engineers 1997; pp. 4–5), where it is stated that "...the basis for the system operation are not explicitly defined. The post processing of the results requires interpretation of the results in order to develop an operation plan that could be used in basic simulation and applied operation."

Many of these hindrances to optimization in reservoir system management are being overcome through ascendancy of the concept of *decision support systems* and dramatic advances in the power and affordability of desktop computing hardware and software. Several private and public organizations actively incorporate optimization models into reservoir system management through the use of decision support systems (Labadie et al. 1989). Incorporation of optimization into decision support systems has reduced resistance to their use by placing emphasis on optimization as a tool controlled by reservoir system managers who bear responsibility for the success or failure of the system to achieve its prescribed goals. This places the focus on providing support for the decision makers, rather than overly empowering computer programmers and modelers.

An example of an optimization model incorporated into a decision support system (DSS) is the *MODSIM* river basin network flow model (Labadie et al. 2000), which is currently being used by the U.S. Bureau of Reclamation for operational planning in the Upper Snake River Basin, Idaho (Larson et al. 1998). The Windows-based graphical user interface (GUI) in *MODSIM* allows the user to create any reservoir system topology by simply *clicking on* various icons and placing system *objects* in any desired configuration on the screen. Data structures embodied in each model object on the screen are controlled by a database management system, with formatted data files prepared interactively and a network flow optimization model automatically executed from the interface. Results of the optimization are presented in useful graphical plots, or even customized reports available through a scripting language included with *MODSIM*. Complex, non-network constraints on the optimization in *MODSIM* are incorporated through an iterative procedure using the embedded *PERL* scripting language. *RiverWare* (Zagona et al. 1998) affords similar DSS functionality with an imbedded preemptive goal programming model providing the optimization capabilities. *RiverWare* has been successfully applied to the TVA system for operational planning (Biddle 2001).

Although lacking a generalized Windows-based graphical user interface, *CALSIM* has been developed by the California Department of Water Resources to allow specification of objectives and constraints in strategic reservoir systems planning and operations without the need for reprogramming (Munevar and Chung 1999). Similar to the use of *PERL* script in *MODSIM*, *CALSIM* employs an English-like modeling language called *WRESL* (Water Resources Engineering Simulation Language) that allows planners and operators to specify targets, objectives, guidelines, constraints, and associated priorities, in ways familiar to them. Simple text file output, along with time series and other data read from relational data bases, are passed to a mixed integer linear programming solver for period by period solution. *CALSIM II* replaces the *DWRSIM* and *PROSIM* models that required continual reprogramming as new objectives and constraints were specified, for coordinated operation of the Federal Central Valley and California State Water Projects. *OASIS* (HydroLogics, Inc.) is a similar modeling package to *CALSIM* that uses an Operations Control Language (OCL) for developing linear programming models for multiobjective analysis of water resource systems.

The explosion of readily available information through the Internet has increased the availability of advanced optimization methods and provided freely accessible software and data resources for successful implementation. Many powerful optimization software packages are available through the Internet, such as from the Optimization Technology Center (Northwestern University and Argonne National Laboratory, Argonne, Illinois) at ⟨http://www.mcs.anl.gov/otc/otc.html⟩. In addition, several spreadsheet software packages available on desktop computers include linear and nonlinear programming solvers in their numerical toolkits. The generalized dynamic programming package *CSUDP* (Labadie 1999) facilitates the use of dynamic programming models, avoiding the need to develop new code for each application. *CSUDP* software is freeware and can be downloaded at ⟨ftp://modsim.engr.colostate.edu/distrib/⟩.

The power and speed of the modern desktop computer have reduced the degree of simplifications and approximations in reservoir system optimization models required in the past, and opened the door to greater realism in optimization modeling. The primacy of the system manager over the model is also emphasized in the incorporation of knowledge-based expert systems into reservoir system modeling which recognize the value of the insights and experience of reservoir system operators. Despite these advances, optimization of the operation of an integrated system of reservoirs still remains a daunting task, particularly with attempts to realistically incorporate hydrologic uncertainties.

Reservoir System Optimization Problem

Objective Function

According to the ASCE Task Committee on Sustainability Criteria (1998), "Sustainable water resource systems are those designed and managed to fully contribute to the objectives of society, now and in the future, while maintaining their ecological, environmental and hydrological integrity." Objective functions used in reservoir system optimization models should incorporate measures such as efficiency (i.e., maximizing current and future discounted welfare), survivability (i.e., assuring future welfare exceeds minimum subsistence levels), and sustainability (i.e., maximizing cumulative improvement over time). Loucks (2000) states that "sustainability measures provide ways by which we can quantify relative levels of sustainability... One way is to express relative levels of sustainability as separate weighted combinations of reliability, resilience and vulnerability measures of various criteria that contribute to human welfare and that vary over time and space. These criteria can be economic, environmental, ecological, and social." The strategy of *shared vision modeling* (Palmer 2000) is useful for enhancing communication among impacted stakeholders and attaining consensus on planning and operational goals.

A generalized objective function for deterministic reservoir system optimization can be expressed as

$$\max_{\mathbf{r}} \ (\text{or} \ \min) \sum_{t=1}^{T} \alpha_t f_t(\mathbf{s}_t, \mathbf{r}_t) + \alpha_{T+1} \varphi_{T+1}(\mathbf{s}_{T+1}) \qquad (1)$$

where $\mathbf{r}_t$=n-dimensional set of control or decision variables (i.e., releases from n interconnected reservoirs) during period t; T=length of the operational time horizon; $\mathbf{s}_t$=n-dimensional state vector of storage in each reservoir at the beginning of period t; $f_t(\mathbf{s}_t, \mathbf{r}_t)$=objective to be maximized (or minimized); $\varphi_{T+1}(\mathbf{s}_{T+1})$ =final term representing future estimated benefits (or costs) beyond time horizon T; and α_t=discount factors for determining present values of future benefits (or costs).

The dynamic nature of this problem reflects the need to represent an uncertain future for sustainable water management; i.e., "... a future we cannot know, but which we can surely influence" (Loucks 2000). The time step t used in this formulation may be hourly, daily, weekly, monthly, or even seasonal, depending on the nature and scope of the reservoir system optimization problem. Hierarchical strategies may also be pursued whereby results from long-term monthly or seasonal studies provide input to more detailed short-term operations over hourly or daily time periods (Becker and Yeh 1974; Divi and Ruiu 1989).

The objective function may be highly nonlinear, such as for maximizing hydropower generation

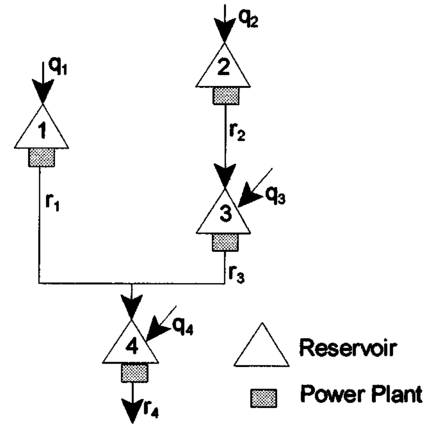

Fig. 1. Example reservoir system configuration

$$f_t(\mathbf{s}_t, \mathbf{r}_t) = \sum_{i=1}^{n} K \cdot e_i(s_{it}, s_{i,t+1}, r_{it}) \cdot \overline{h_{it}}(s_{it}, s_{i,t+1}) \cdot r_{it} \cdot \Delta t_{it} \qquad (2)$$

where e_i=overall powerplant efficiency at reservoir i as a function of average head and discharge during period t; $\bar{h}_{it}$=average head as a function of beginning and ending period storage levels (calculated from the reservoir mass balance or system dynamics equation), as well as possibly the discharge if tailwater effects are included; K=unit conversion factor; and Δt_{it}=number of on-peak hours related to the load factor for powerplant i. This is a highly nonconvex function characterized by many local maxima (Tauxe et al. 1980), and may be discontinuous and nondifferentiable if loading of individual turbines in the powerplant is considered. Other objective functions related to vulnerability criteria may attempt to minimize deviations from ideal target storage levels, water supply deliveries, discharges, or power capacities. If economic benefit and cost estimates are available for these uses, then the objective may be to maximize total expected net benefits from operation of the system, but with consideration of long-term sustainability.

Constraints

The system dynamics or state-space equations are written as follows, based on preservation of conservation of mass throughout the system:

$$\mathbf{s}_{t+1} = \mathbf{s}_t + \mathbf{C}\mathbf{r}_t + \mathbf{q}_t - \mathbf{l}_t(\mathbf{s}_t, \mathbf{s}_{t+1}) - \mathbf{d}_t \quad (\text{for } t=1,...,T) \qquad (3)$$

where $\mathbf{s}_t$=storage vector at the beginning of time t; $\mathbf{q}_t$=inflow vector during time t; $\mathbf{C}$=system connectivity matrix mapping flow routing within the system; $\mathbf{l}_t$=vector combining spills, evaporation, and other losses during time t; and $\mathbf{d}_t$=required demands, diversions, or depletions from the system. In some formulations, diversions are treated as decision variables and included in the objective function as related to benefits of supplying water. Accurate calculation of evaporation and other water losses in the term $\mathbf{l}_t(\mathbf{s}_t, \mathbf{s}_{t+1})$ creates a set of nonlinear implicit equations in $\mathbf{s}_{t+1}$ which can be difficult to evaluate and constitute a nonconvex feasible set. Initial storage levels $\mathbf{s}_1$ are assumed known and all flow units in Eq. (3) are expressed in storage units per unit time.

Spatial connectivity of the reservoir network is fully described by the routing or connectivity matrix $\mathbf{C}$. For the example reservoir system of Fig. 1, the connectivity matrix is

$$\mathbf{C}=\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 0\\ 0 & +1 & -1 & 0\\ +1 & 0 & +1 & -1\end{bmatrix}$$

Additional state variable nodes with zero storage capacity may represent nonstorage locations where inflows and diversions occur. For more complex system configurations that are nondendritic, such as bifurcating systems and off-stream reservoirs, a more complex link-node connectivity matrix is required. Lagged routing of flows can be considered by replacing the term $\mathbf{Cr}_t$ in Eq. (3) with $\Sigma_{\tau=0}^{k}\mathbf{C}_\tau\mathbf{r}_{t-\tau}$, where elements of the routing matrices $\mathbf{C}_\tau$ may be fractions representing lagging and attenuation of downstream releases.

Explicit lower and upper bounds on storage must be assigned for recreation, providing flood control space, and assuring minimum levels for dead storage and powerplant operation.

$$\mathbf{s}_{t+1,\min}\leqslant\mathbf{s}_{t+1}\leqslant\mathbf{s}_{t+1,\max}\quad(\text{for}\ \ t=1,...,T)\tag{4}$$

Limits on reservoir releases are specified as

$$\mathbf{r}_{t,\min}\leqslant\mathbf{r}_t\leqslant\mathbf{r}_{t,\max}\quad(\text{for}\ \ t=1,...,T)\tag{5}$$

These limits maintain minimum desired downstream flows for water quality control and fish and wildlife maintenance, as well as protection from downstream flooding. In some cases, it may be necessary to specify these limits as functions of head where allowable discharges depend on reservoir storage levels. Additional constraints may be imposed on the *change* in release from one period to the next to provide protection from scouring of downstream channels. When evaluating long term historical or synthetic hydrologic sequences, or multiple short-term sequences, difficulties may arise in finding feasible solutions that satisfy these constraints. In these cases, it may be necessary to relax these as explicit constraints and indirectly consider them through use of weighted penalty terms on violation of these constraints in the objective function.

Other constraints may represent alternative objectives that must be maintained at desired target levels $\boldsymbol{\varepsilon}$:

$$\overline{\mathbf{f}}(\mathbf{s},\mathbf{r})\geqslant\boldsymbol{\varepsilon}\tag{6}$$

Example targets might include annual water supply requirements or power capacity maintenance. These targets may be adjusted parametrically to compute tradeoff relations between the primary objective of Eq. (1) and secondary objectives as a means of providing multiple objective solutions (Cohon 1978).

The optimization model defined in Eqs. (1)–(6) is challenging to solve since it is dynamic, potentially nonlinear, and nonconvex; and large-scale. In addition, unregulated inflows, net evaporation rates, hydrologic parameters, system demands, and economic parameters should often be treated as random variables, giving rise to a complex large-scale, nonlinear, stochastic optimization problem. In this formulation, it is assumed that calibration and verification studies have been carried out to assure the model is capable of reasonably reproducing historical energy production, storage levels, and flows throughout the system. This review explores several solution strategies, including implicit stochastic optimization, explicit stochastic optimization, real-time optimal control with forecasting, and heuristic programming methods. For more detailed treatment of these topics, the reader is referred to a number of important books written over the years dealing with optimization of water resource systems in general, and optimal operation of reservoirs in particular. These include: Maass et al. (1962); Hall and Dracup (1970); Buras (1972); Loucks et al. (1981); Mays and Tung (1992); Wurbs (1996); and ReVelle (1999).

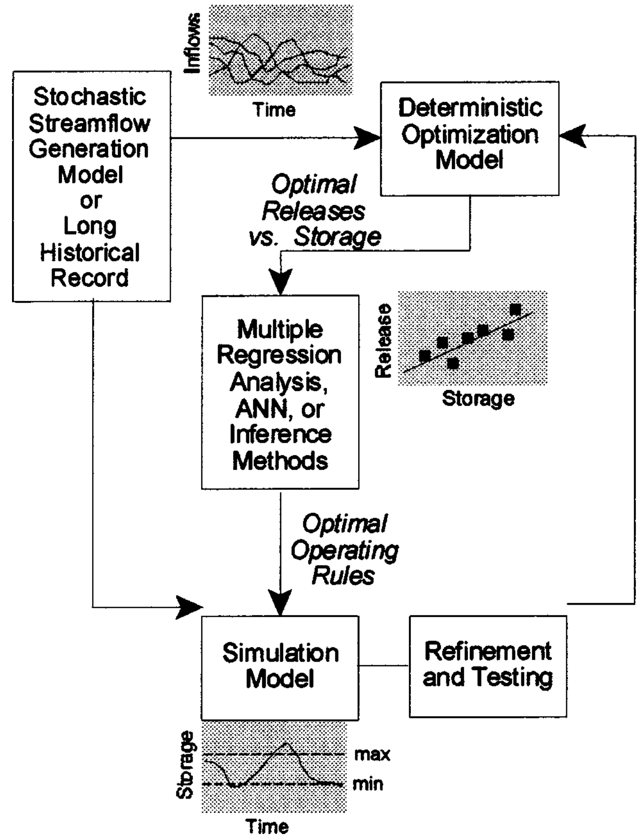

Fig. 2. Implicit stochastic optimization (ISO) procedure

Implicit Stochastic Optimization

The solution of Eqs. (1)–(6) may be accomplished by implicit stochastic optimization (ISO) methods, also referred to as Monte Carlo optimization, which optimize over a long continuous series of historical or synthetically generated unregulated inflow time series, or many shorter equally likely sequences (Fig. 2). In this way, most stochastic aspects of the problem, including spatial and temporal correlations of unregulated inflows, are implicitly included and deterministic optimization methods can be directly applied. The primary disadvantage of this approach is that optimal operational policies are unique to the assumed hydrologic time series. Attempts can be made to apply multiple regression analysis and other methods to the optimization results for developing seasonal operating rules conditioned on observable information such as current storage levels, previous period inflows, and/or forecasted inflows. Unfortunately, regression analysis may result in poor correlations that invalidate the operating rules, and attempting to infer rules from other methods may require extensive trial and error processes with little general applicability.

Linear Programming Models

Since ISO models can be extremely large-scale, covering a lengthy time horizon, it is critical that only the most efficient optimization methods are applied. One of the most favored optimization techniques for reservoir system models is the simplex method of linear programming and its variants (Nash and Sofer 1996). These models require all relations associated with Eqs. (1)–(6) to be linear or linearizable. The advantages of linear programming (LP) include: (1) ability to efficiently solve large-scale problems; (2) convergence to global optimal solutions; (3) initial solutions not required from the user; (4) well-developed duality theory for sensitivity analysis; and (5) ease of problem setup and solution using readily available, low-cost LP solvers. Recent alternatives to the simplex method, such as the affine scaling and interior projection methods (Terlaky 1996), are particularly attractive for solving extremely large-scale problems.

Hiew et al. (1989) applied ISO using LP to the eight-reservoir Colorado-Big Thompson (C-BT) project in northern Colorado. Use of a 30 year historical hydrologic record of monthly unregulated inflows to the system resulted in a linear programming problem with 12,613 variables and 5,040 constraints. Multiple regression analysis was applied to the LP model results to produce optimal lag-one storage guide curves:

$$\mathbf{s}_{t+1}=\overline{\mathbf{A}}\mathbf{s}_t^*+\overline{\mathbf{B}}\mathbf{q}_{t-1}+\overline{\mathbf{c}} \tag{7}$$

where $\mathbf{s}_t^*$=optimal storage levels obtained from the linear programming solution; $\mathbf{q}_t$=observed hydrologic inflows; and correlation matrices $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$ and vector $\overline{\mathbf{c}}$ are calculated from multiple regression analysis performed on the LP results. Coefficients of determination obtained from this analysis ranged from 0.795 to 0.996 for the larger reservoirs, with the remaining reservoirs either small or with water levels only allowed to vary a few feet per year. Simulation of the system operations using the optimal storage guide curves of Eq. (7) confirmed their validity. This study was successful because of the ability of linear models to accurately represent the system behavior, along with the fortunate calculation of high correlation coefficients obtained from the multiple regression analysis. For other systems, these advantages may not be in evidence.

Other extensions of linear programming into binary, integer, and mixed-integer programming may be valuable for representing highly nonlinear, nonconvex terms in the objective function and constraints (e.g., Trezos 1991), but these methods are considerably less efficient computationally and would likely be intractable for use in ISO. Needham et al. (2000) applied mixed integer linear programming to deterministic flood control operations in the Iowa and Des Moines Rivers, but noted the potential for excessive computer times when extended to stochastic evaluation. This study came to the rather counterintuitive conclusion that coordinated operation of reservoir systems does not necessarily improve performance, which stands in stark contrast with other studies that have shown just the opposite (e.g., Shim et al. 2002).

Piecewise linear approximations of nonlinear functions are often used in *separable programming* applications and solved with various extensions of the simplex method, although problem size can become excessive in some cases. Functions of more than one variable can be approximated using multilinear interpolation methods over a multidimensional grid. For minimization problems, these functions must be convex; otherwise, more time consuming *restricted basis entry* simplex algorithms must be applied which fail to guarantee convergence to global optima. Crawley and Dandy (1993) applied separable programming to the multi-reservoir Metropolitan Adelaide water supply system in Australia.

Network Flow Optimization Models

It is evident from Fig. 1 that an interconnected reservoir system can be represented as a network of nodes and links (or arcs).

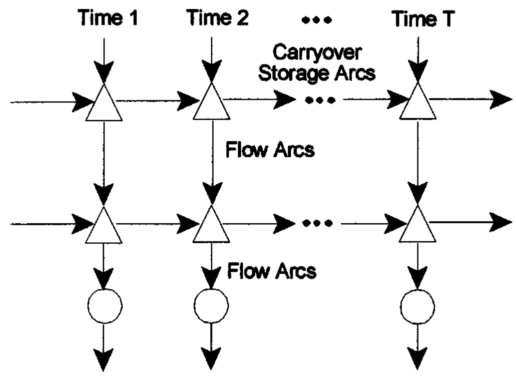

Fig. 3. Illustration of dynamic network showing carryover storage arcs

Nodes are storage or nonstorage points of confluence or diversion, and links represent reservoir releases, channel or pipe flows, carryover storage, and evaporation and other losses. If all relations in Eqs. (1)–(5) are linear, then the following dynamic, minimum cost network flow problem results:

$$\text{minimize} \sum_{t=1}^{T}\sum_{\ell\in A} c_{\ell t}x_{\ell t} \tag{8}$$

subject to

$$\sum_{j\in\mathbf{O}_i} x_{jt}-\sum_{k\in\mathbf{I}_i} x_{kt}=0 \quad (\text{for all } i\in\mathbf{N};\quad \text{for all } t=1,\ldots,T) \tag{9}$$

$$l_{\ell t}\leq x_{\ell t}\leq u_{\ell t} \quad (\text{for all } \ell\in\mathbf{A};\quad \text{for all } t=1,\ldots,T) \tag{10}$$

where $\mathbf{A}$=set of all arcs or links in the network; $\mathbf{N}$=set of nodes; $\mathbf{O}_i$=set of all links originating at node i (i.e., outflow links); $\mathbf{I}_i$=set of all links terminating at node i (i.e., inflow links); $x_{\ell t}$=flow rate in link ℓ during period t; $c_{\ell t}$=costs, weighting factors, or priorities per unit of flow rate in link ℓ during period t; and $l_{\ell t}$ and $u_{\ell t}$=lower and upper bounds, respectively, on flow in link ℓ.

Fig. 3 illustrates a fully dynamic network where the horizontal arcs represent carryover storage (i.e., $\mathbf{s}_t$) in the same physical reservoir from one period to the next, whereas the vertical arcs are flows, releases, and diversions (i.e., $\mathbf{r}_t$) during the current period. Eqs. (8)–(10) define a *pure* network formulation where all network data can be represented by a set of arc parameters $[l_{\ell t},u_{\ell t},c_{\ell t}]$. For fully circulating networks, additional artificial nodes and links must be added for satisfying overall mass balance throughout the entire network. Comparative studies by Kuczera (1993) and Ardekaaniaan and Moin (1995) have shown the dual coordinate ascent *RELAX* algorithm (Bertsekas and Tseng 1994) to be the most efficient network solver, as compared to primal-based algorithms and variations on the out-of-kilter method (Ford and Fulkerson 1962).

Several network algorithms allow designation of node supply and demand [i.e., entry of values other than zero on the *right-hand side* of Eq. (9)] without requiring specification of artificial nodes and links, although this is only possible when no demand shortages occur. For so-called *networks-with-gains*, Eq. (9) must be adjusted with coefficients not equal to −1, 0, or +1 to allow for channel losses, evaporation losses, and return flows. Further extensions into *generalized networks* allow inclusion of *side constraints* [i.e., Eq. (6)] that violate the pure network structure. All of these deviations from the pure network format exact a computational price. In spite of this, Sun et al. (1995) claim that the generalized network solver is ''11–17 times faster'' than solution by a state-of-the-art revised simplex algorithm. For pure network problems, this speedup factor may increase to more than two orders of magnitude, while requiring significantly less computer memory. Hsu and Cheng (2002) applied a similar generalized network flow optimization model for long-term supply-demand analysis in a river basin in northern Taiwan. Although the results show improvement over previous simulation studies, the perfect foreknowledge assumptions in the deterministic evaluation and lack of development of conditional operating rules diminish the value of the study.

Fredericks et al. (1998) show that many aspects of networks-with-gains and generalized networks can be solved through successive solution of pure network problems with adjusted arc parameters, ultimately converging to solution of the original problem. This approach may be more efficient if the speedup factor for pure networks is compensated by the need for a few iterations to achieve convergence. Labadie and Baldo (2001) report an additional advantage of the successive solution approach that avoids the need for computationally expensive unit priority separation procedures (Israel and Lund 1999) for correct allocation of flows according to water rights and other prioritizing mechanisms. The successive solution procedure has a further advantage of allowing consideration of non-network side constraints and the final convergent solution still provides node and arc prices as dual variables that are useful for sensitivity analysis.

Lund and Ferreira (1996) applied a fully dynamic network flow algorithm HEC-PRM to the mainstem Missouri River Reservoir system. Although the network itself is not large (i.e., six storage nodes and six intermediate flow nodes), the system is optimized in monthly time steps over a 90 year historical period, resulting in an immense dynamic network. The objective function is approximated by convex, piecewise linear penalty functions characterized through specification of multiple links connecting two nodes, with bounds and unit costs defined by flow limits and slopes of each linear piece. In this study, ISO procedures of performing regression analyses on operating rules that condition optimal releases on total system storage resulted in poor correlation coefficients. Empirical trial-and-error processes are invoked, which ultimately result in reasonable rules when evaluated using a simulation model for the system operation.

Nonlinear Programming Models

Many reservoir system optimization problems cannot be realistically modeled using piecewise linearization, and must be attacked directly as nonlinear programming problems, particularly with inclusion of hydropower generation in the objective function and/or constraints. Nonlinear programming (NLP) algorithms generally considered the most powerful and robust are: (i) successive (or sequential) linear programming (SLP); (ii) successive (or sequential) quadratic programming (SQP) (or projected Lagrangian method); (iii) augmented Lagrangian method [or method of multipliers (MOM)]; and (iv) the generalized reduced gradient method (GRG). All require that the functions in Eqs. (1)–(6) are differentiable, which may be problematic in some cases, particularly for hydropower systems. Explicit calculation of derivatives is unnecessary, however, with application of automatic differentiation methods (Sinha and Bischof 1998).

Hiew (1987) performed a comprehensive comparative evaluation of the SLP, GRG, and a feasible direction form of SQP for hydropower systems of up to seven reservoirs, and concluded that the SLP method was by far the most efficient (by up to an order of magnitude in computational speed) among the various nonlinear programming algorithms. Grygier and Stedinger (1985) also concluded that SLP was the most efficient of the mathematical programming algorithms evaluated. In SLP, all nonlinear functions are linearized around an initial or nominal solution using the first two terms of the Taylor series expansion. Successive solutions are confined to specified *trust regions* or step bounds to avoid instabilities in convergence. Solution of the resulting linear programming problem then provides the basis for relinearization of the nonlinear functions, with the step bounds appropriately reduced as the process converges. A disadvantage of SLP is that ''although intuitively appealing and popular because of the availability of efficient linear programming solvers, ...(the method is)... not guaranteed to converge'' (Bazarra et al. 1993).

Martin (1983) applied the SLP method to the Arkansas-White-Red River system of Texas, noting that the linearized subproblems could be efficiently solved using a minimum cost network flow solver. A fully dynamic network model is used, but solutions are generated over a 5 year period by a moving overlapping window approach that finds optimal dynamic solutions over successive 2 year periods. The rationale for limiting the operational interval in each optimization is based on the likelihood of the storage projects in the system filling based on carryover storage capability. Once a project fills, all memory of previous operations is lost; hence, fully dynamic solutions over extremely long time horizons, such as in Lund and Ferreira (1996), may not be necessary. Barros et al. (2003) applied the SLP technique to the Brazilian hydropower system, one of the largest in the world. This study also confirmed the excellent performance of SLP, both in accuracy and computational efficiency.

Successive quadratic programming (SQP) relies on the computational efficiency of modern quadratic programming (QP) algorithms and the ability of quadratic expansions to better approximate nonlinear functions than linear relations. Instead of linearizing the objective function, a quadratic approximation is performed on the Lagrangian function for the problem, although the constraints continue to be linearized. Successive quadratic approximations converge to a Karush-Kuhn-Tucker (KKT) point satisfying the necessary conditions for an optimal solution (Bazarra et al. 1993). To avoid a potentially time consuming solution of a large-scale QP problem over many time intervals, Arnold et al. (1994) proposed a procedure that takes advantage of the special structure of reservoir system optimization problems and provides a highly efficient solution algorithm for the QP problem. Numerical effort grows only linearly with the length of the operational horizon T since the method decomposes the problem into subproblems for each time step. Without this modification, it is unlikely that SQP is suitable for ISO of reservoir systems due to exponential growth of computer time with the number of time steps.

Tejada-Guibert et al. (1990) applied SQP to a five-reservoir portion of the Central Valley Project (CVP) of California using *MINOS* (Murtagh and Saunders 1987). The objective function includes nonlinear terms representing operating costs avoided and projects dependable hydropower capacity for each powerplant. Constraints in the form of Eq. (6) include nonlinear functions of energy production per unit release. A 3 year optimization over monthly time steps resulted in a problem with 1,122 variables and 1,764 constraints. The authors note that computer execution times increase approximately to the square of the length of the operational period, which does not bode well for application of ISO over long time periods. Barros et al. (2003) also applied SQP

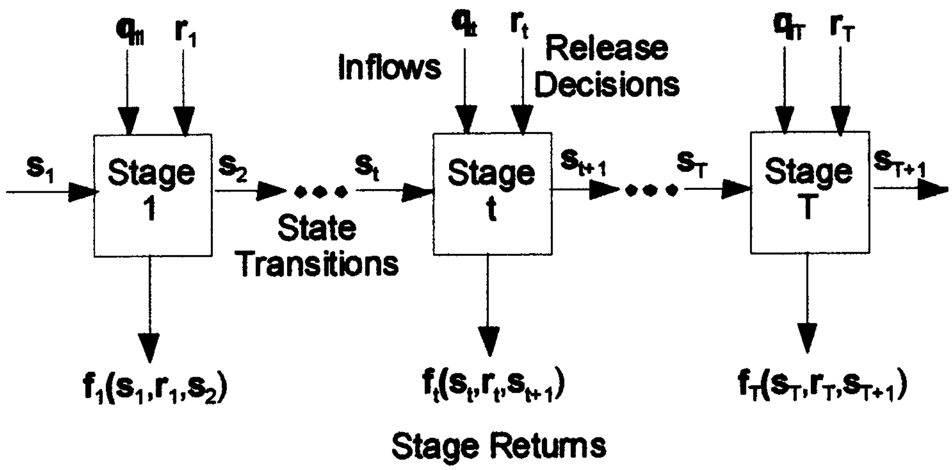

Fig. 4. Illustration of reservoir system optimization as sequential decision process

using a newer version of *MINOS* (Murtagh and Saunders 1995) to the large-scale Brazilian hydropower system, and compared performance with SLP. Although it was found to produce more accurate results, computational requirements restricted its use to optimization over limited real-time forecast horizons rather than for implicit stochastic optimization over lengthy time horizons.

Arnold et al. (1994) compared SQP with the augmented Lagrangian method or method of multipliers (MOM). MOM uses a Lagrangian function similar to SQP, but augmented with exact penalty terms. The original constrained nonlinear optimization problem is replaced with a sequence of easier-to-solve unconstrained nonlinear optimization problems. Arnold et al. (1994) applied SQP and MOM to the four-reservoir Zambezi River system in southern Africa over a 2 year period in monthly time steps. The model includes realistic nonlinear terms for hydropower production and evaporation calculations, resulting in a large-scale, dynamic optimization problem with nonlinear objective function and constraints. Results show that MOM converged more rapidly than SQP, but to a somewhat less accurate solution.

The generalized reduced gradient (GRG) method is essentially a constrained gradient search technique that solves a *reduced* optimization problem with respect to the independent decision variables $\mathbf{r}_t$. The storage state variables $\mathbf{s}_t$ are easily determined from Eq. (3) for a given set of decision variables $\mathbf{r}_t$. The *MINOS* optimization package (Murtagh and Saunders 1995) invokes GRG for nonlinear programming problems with linear constraints. Although the method can theoretically be applied to nonlinear constraints through successive linearization, it appears to be less efficient in this case. Unver and Mays (1990) applied GRG to optimal flood control in the Highland Lakes system of the Lower Colorado River Basin, Texas. GRG had to be combined with penalty function methods similar to MOM to properly treat the inequality constraints on the dependent reservoir storage variables $\mathbf{s}_t$.

Peng and Buras (2000) also applied GRG (using *MINOS*) within an ISO scheme to the five major upstream lakes in the West Branch Penobscot River, Maine. Similar to the approach of Martin (1983), the GRG optimization is performed in monthly time steps over a moving 12 month forecast window. A synthetic streamflow generation model produces multiple, equally likely inflow sequences over the next 12 months, starting from the current month, with the SQP optimization performed for each sequence. Although operational decisions are obtained for each month, only the current month decisions are implemented. Consistent with the problem of any ISO application, since unique decisions are generated for each synthetic inflow sequence, the decisions are represented as randomized release rules that are difficult to implement.

Discrete Dynamic Programming Models

Next to linear programming, dynamic programming has been the most popular optimization technique applied to water resources planning and management in general, and reservoir operations in particular (Yakowitz 1982). Dynamic programming (DP) effectively exploits the sequential decision structure of reservoir system optimization problems (Fig. 4). As originally developed in its general form by Bellman (1957), DP decomposes the original problem into subproblems that are solved sequentially over each stage (i.e., time period). This represents a significant advantage for ISO since computational effort increases only linearly with the number of stages, whereas most of the previous methods display exponential increases. The earliest application of ISO applied dynamic programming to a single reservoir operational problem (Young 1967). In its discrete form, DP overcomes difficulties with functional relationships in the objective and constraints that are nonlinear, nonconvex, and even discontinuous. It is also more readily extensible to explicit stochastic optimization problems, and existence of constraints such as Eqs. (4) and (5) actually improve solution efficiency, in contrast with the other methods discussed.

Solution of Eqs. (1)–(5) involves calculating an optimal return or *cost-to-go* function $F_t(\mathbf{s}_t)$ representing the maximum return (or minimum cost) accumulated from the current period (stage) t through the final period T, conditioned on a given initial storage state vector $\mathbf{s}_t$. Bellman's *principle of optimality* (Bellman 1957) states that: *no matter what the initial stage and state of a Markovian decision process, there exists an optimal policy from that stage and state to the end*. For all discrete combinations of $\mathbf{s}_t$, the function $F_t(\mathbf{s}_t)$ is optimized recursively over each time period in a (usually) backwards sequence for $t=T,\ T-1,...,1$:

$$F_t(\mathbf{s}_t)=\max_{\mathbf{r}_t}\ (\text{or}\ \ \min)\,[\alpha_t f_t(\mathbf{s}_t,\mathbf{r}_t)+F_{t+1}(\mathbf{s}_{t+1})] \qquad (11)$$

subject to Eqs. (3)–(5). Recursive calculations are initiated with

$$F_{T+1}(\mathbf{s}_{T+1})=\alpha_{T+1}\varphi_{T+1}(\mathbf{s}_{T+1}) \qquad (12)$$

DP takes advantage of the temporal separability of the problem defined by Eqs. (1)–(5), although inclusion of Eq. (6) invalidates this separability. Consideration of the latter requires additional state variables in the formulation or application of Lagrange multiplier techniques (Dreyfus and Law 1977). Another advantage of DP is the calculation of flexible *feedback* or *closed-loop* optimal policies $\mathbf{r}_t^*(\mathbf{s}_t)$ conditioned on the current system state $\mathbf{s}_t$. Except for optimal control theory, the development of optimal feedback policies is unique to dynamic programming. Optimal storage guide curves $\mathbf{s}_{t+1}^*(\mathbf{s}_t)$ can also be calculated, which may be more useful to reservoir system operators than optimal release policies.

Discrete dynamic programming increments reservoir storage levels in the vector $\mathbf{s}_t$ into a finite number of levels and then performs conditional optimization in Eq. (11) over all possible discrete combinations of storage levels. Global optimality, in a discrete sense, is assured if the optimization in Eq. (11) is performed via exhaustive enumeration over all discrete combinations of releases. Since this optimization is performed conditionally on all discrete combinations of storage vector $\mathbf{s}_t$, the specter of the *curse of dimensionality* [a term originally coined by Bellman (1961)] arises. Assuming an average of *m* discretization levels for each of *n* reservoirs, computational time and storage requirements are proportional to m^n. For system dimensions beyond three connected reservoirs, rapid access memory requirements exceed the capacity of modern computing hardware.

Various modifications have been performed on the original DP formulation to mollify the *curse of dimensionality* of discrete dynamic programming. These include: (i) coarse grid/interpolation techniques; (ii) dynamic programming successive approximations (DPSA); and (iii) incremental dynamic programming (IDP) or discrete differential dynamic programming (DDDP). Coarse grid/interpolation methods attempt to reduce the intensive core memory and computational requirements of evaluating and storing the optimal return or cost-to-go function $F_t(\mathbf{s}_t)$ for all discrete combinations of the vector $\mathbf{s}_t$ by using larger discretization intervals. Solution accuracy is retained by interpolating the function over the coarser grid structure. This approach was first suggested by Bellman (1957), and later extended by Johnson et al. (1993) and others to sophisticated interpolation methods using high order piecewise polynomial functions. Although these methods alleviate the dimensionality problem, they fail to vanquish it completely.

Bellman and Dreyfus (1962) originally suggested the dynamic programming successive approximations (DPSA) technique, later generalized by Larson (1968). DPSA decomposes the multidimensional problem into a sequence of one-dimensional problems by optimizing over one state variable at a time, with all other state variables maintained at given current values. This requires that the state dynamics equations [Eq. (3)] must be *inverted* to explicitly solve for releases $\mathbf{r}_t$ as a function of specified beginning and ending storage levels $\mathbf{s}_t$, $\mathbf{s}_{t+1}$. This results in accurate calculation of evaporation losses without the need for iterative procedures, and allows for the development of optimal storage guide curves.

Although convergence to a global optimum is guaranteed for convex problems, convergence to even local optima with DPSA is not assured for nonconvex problems. Collins (1977) proposed a *two-at-a-time* DPSA method that adjusts the state variables in overlapping pairs. DPSA and its extensions (e.g., IDPSA, which essentially combines IDP and DPSA by solving over one state at a time, but confining each state to an incremental corridor) has been applied to many multiple reservoir systems, including the Central Valley Project (Yeh and Trott 1972) and the Tennessee Valley Authority (Giles and Wunderlich 1981). DPSA was applied by Shim et al. (2002) for real-time flood control operations in the Han River Basin, Korea. An iterative approach involving successive solutions of the DPSA algorithm with updated estimates of river reach routing coefficients allowed full incorporation of routing in the optimization over hourly time steps. DPSA was also applied by Yi et al. (2003) for optimal hourly scheduling of hydropower units in the Lower Colorado River Reservoir System. Comparison of the DPSA solutions with a large-scale mixed integer programming (MIP) formulation provided comparable accuracy, but at a fraction of the computer execution time required by the MIP model.

Incremental dynamic programming (IDP) was first introduced by Larson (1968), and applied by Hall et al. (1969) to a portion of the Central Valley Project in northern California. Discrete differential dynamic programming (DDDP) was later offered by Heidari et al. (1971), but closely resembles the IDP technique. These algorithms address the dimensionality problem by restricting the state-space to a corridor around a current given solution $\mathbf{s}_t^{(k)}$. For each state variable, only three discrete values are allowed for a specified storage increment Δs_i :$[s_{it}^{(k)}-\Delta s_i, s_{it}^{(k)}, s_{it}^{(k)}+\Delta s_i]$. If new solution trajectories are within the boundaries of the corridor, the optimum has been found. Otherwise, a new corridor is defined around the new solution and the process repeats. The computational effort for each solution is now proportional to 3^n.

Difficulties with IDP or DDDP methods are: (1) as with DPSA, the inverted form of the state equations [Eq. (3)] must be used to avoid interpolation problems over the restricted corridor; (2) the method is highly sensitive to initially assumed storage trajectories $\mathbf{s}_t^{(0)}$; and (3) discretization intervals Δs_i must be carefully selected to provide accurate solutions at reasonable computational expense. One attractive approach is to initially select large values, which can then be refined as the neighborhood of the optimum trajectory is approached. Unlike DPSA, convergence to a discrete local optimum, under reasonably mild assumptions, is guaranteed. As with DPSA, global optima are only attainable for convex problems. The generalized dynamic programming software package *CSUDP* (Labadie 1999) employs a strategy, originally suggested by Nopmongcol and Askew (1976), whereby solutions are initiated with the DPSA technique, which rapidly converge to the neighborhood of the optimum. The IDP/DDDP method then either further refines this solution or confirms that it is a true (discrete) local optimum.

Karamouz et al. (1992) applied discrete dynamic programming to a multiple site reservoir system in the Gunpowder River Basin near Baltimore. A total of 1,500 months of multisite, synthetic streamflow data were input into the discrete DP model for ISO. A linear operating rule structure similar to Eq. (7) was adopted, with the authors noting that more complex nonlinear rules have little advantage. To overcome difficulties in the multiple regression analysis when correlation coefficients are low, successive solutions are bounded to be within a certain percentage of the optimal operating rules found from the previous implicit stochastic DP run. With each successive iteration, correlation coefficients for the operating rules increase until the process terminates with consistent operating rules. This process performed well for the two-site system considered by the authors, but extensions to larger reservoir systems may be difficult.

Differential Dynamic Programming Models

Jacobson and Mayne (1970) developed differential dynamic programming (DDP) to alleviate dimensionality difficulties in DP through use of analytical solutions rather than resorting to discretization of the state-space. Murray and Yakowitz (1979) extended this approach to more realistic constrained problems, but differentiability of the objective function and constraints is still required. DDP can be thought of as an SQP methodology specifically designed to exploit the sequential decision structure of problems such as reservoir system optimization. This implies that computational effort increases approximately linearly with the number of stages, making it a feasible strategy for ISO. In addition, Sen and Yakowitz (1987) contend that DPSA and IDP methods can only hope for linear rates of convergence, whereas superlinear or even quadratic convergence similar to Newton-type methods can be expected with DDP.

Explicit analytical solutions can be obtained for Eq. (11) if the objective function is quadratic and any inequality constraints [i.e., Eqs. (4)–(6) for the deterministic case] are relaxed (Dreyfus and Law 1977). Under these assumptions, the DP optimal return or cost-to-go function $F_t(\mathbf{s}_t)$ is a quadratic function of $\mathbf{s}_t$ and is easily represented analytically rather than numerically as in standard DP. For the constrained case, Murray and Yakowitz (1979) propose the solution of a constrained quadratic programming (QP) problem derived by approximating the nonlinear objective function $f_t(\mathbf{s}_t,\mathbf{r}_t)$ using the first three terms of the Taylor series expansion around a current nominal state trajectory $\mathbf{s}_t^{(k)}$ at iteration k. Sen and Yakowitz (1987) suggest replacing the Hessian matrix of second partial derivatives in the Taylor series expansion with first order approximations based on quasi-Newton updates that guarantee convexity of the QP problem.

Jones et al. (1986) applied the DDP approach to ISO of the Mad River system in northern California. A total of 101 sets of 64 years of stochastically generated monthly inflows were input to the DDP algorithm for minimizing downstream water deficits. The authors note that a linear programming formulation of the same problem required 16 times the computer processing time as the DDP algorithm. Rather than applying regression analysis to infer optimal long-term release policies, release rules were conditioned on ranges of current period inflow and storage with exceedence probabilities of 0.95.

Discrete-Time Optimal Control Theory

All of the methods discussed thus far are categorized as mathematical programming techniques. Optimal control theory (OCT) represents a different approach to optimization, but in its discrete-time form, shares many similarities with mathematical programming. Modern optimal control theory has its origins with Pontryagin's maximum principle (Pontryagin et al. 1962), which was originally derived for optimal control of dynamic systems governed by differential equations under control constraints. For continuous-time problems, the maximum (or minimum) principle states that a particular decision and state trajectory is optimal if there exists an adjoint trajectory such that the Hamiltonian function is maximized (or minimized). The Hamiltonian is formed by appending the system dynamics equations to the original objective function using *costate* or *adjoint* variables, which in mathematical programming parlance, are Lagrange multipliers. This results in a difficult *two-point boundary value* problem, particularly with inclusion of state-space constraints. Pontryagin et al. (1962) showed that constraints on the decision variables can be explicitly maintained in the optimization, although certain convexity conditions are required in the discrete-time case.

Extensions of discrete OCT to reservoir system optimization problems governed by difference equations [i.e., Eq. (3)] strongly resemble the MOM method of nonlinear programming, but with important differences. Similar to MOM, an augmented Lagrangian function is formulated, but with Lagrange multipliers only associated with the system dynamics equations and penalty terms assigned to the state-space constraints [Eq. (4)]. Formulations attempting to include the state-space constraints with Lagrange multipliers result in complex *corner* or *jump conditions* that are difficult to evaluate (Grygier and Stedinger 1985).

Hiew (1987) compared the performance of the OCT algorithm with SLP, SQP, and the GRG nonlinear programming algorithms for a multireservoir hydropower system. OCT outperformed all of the other methods in computational speed, was comparable in solution accuracy, and least sensitive to the initial solution as compared with other methods. Papageorgiou (1988) applied OCT to multireservoir systems optimization and found computer time requirements increased only linearly with the number of reservoirs.

Mizyed et al. (1992) applied OCT to the large 11-reservoir system in the Mahaweli Valley of Sri Lanka. The nonlinear objective maximizes firm energy production subject to irrigation demand requirements. A 32 year historical record of monthly inflows was used for ISO, but with irrigation demands set at mean monthly values. Multiple regression analysis was applied to the OCT optimization results for determining optimal regression coefficients in an operating rule similar to Eq. (7). In this case, matrices **A**, **B** were assumed to be diagonal (i.e., no spatial cross correlations considered), although additional regression terms were added for total storage, total inflow, and total irrigation demand. This resulted in reasonable coefficients of determination r^2 varying from 0.6 to 0.9 for all reservoirs.

Explicit Stochastic Optimization

Explicit stochastic optimization (ESO) is designed to operate directly on probabilistic descriptions of random streamflow processes (as well as other random variables) rather than deterministic hydrologic sequences. This means that optimization is performed without the presumption of perfect foreknowledge of future events. In addition, optimal policies are determined without the need for inferring operating rules from results of the optimization (Fig. 5). Unfortunately, ESO techniques as applied to multireservoir systems are more computationally challenging than ISO, as recognized early by Roefs and Bodin (1970).

For ESO, Eq. (1) is now formulated as

$$\max_{\mathbf{r}} \ (\text{or} \ \min) \ \mathbf{E}_{\mathbf{q}}\left[\sum_{t=1}^{T} \alpha_t f_t(\mathbf{s}_t,\mathbf{r}_t,\mathbf{q}_t)+\alpha_{T+1}\varphi_{T+1}(\mathbf{s}_{T+1})\right] \tag{13}$$

where $\mathbf{E}$=statistical expectation operator. Alternative formulations based on Markov decision theory consider infinite time horizons where final terms defining future benefits or costs are not required (Bertsekas 1987). The goal here is to determine long-term (seasonally) stationary optimal operational policies.

Since inflows $\mathbf{q}_t$ are now regarded as random variables, storage levels calculated via Eq. (3) are also random, meaning that Eqs. (4) and (5) must be expressed probabilistically:

$$\Pr[s_{i,t+1}\geqslant s_{i,t+1,\min}]\geqslant(1-\alpha) \tag{14}$$

$$\Pr[s_{i,t+1}\leqslant s_{i,t+1,\max}]\geqslant(1-\beta) \quad (\text{for} \ \ i=1,...,n; \ \ t=1,...,T) \tag{15}$$

where α and β=desired levels of *risk* of violating these constraints, which may vary by season. In this case, unregulated inflows are assumed the dominant source of uncertainty and can be represented by appropriate probability distributions. These may

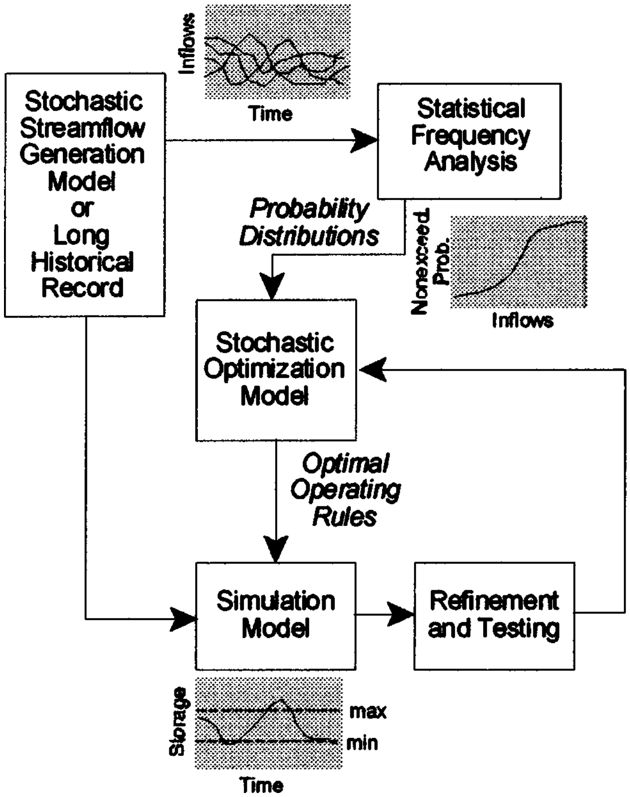

Fig. 5. Explicit stochastic optimization (ESO) procedure

be parametric or nonparametric based on frequency analysis. Other random variables that may be defined include economic parameters in the objective function, demands, and climatological variables impacting net evaporation and other losses. Unregulated inflows may be highly correlated spatially and/or temporally. For short-term operational problems, inflows may be generated from forecasting models, in which case the primary source of uncertainty is the forecast error.

Chance-Constrained Programming Models

The cumulative probability distribution function for independent unregulated inflows to site i during period t is

$$F_{it}(x)=\Pr[q_{it}\leqslant x] \tag{16}$$

Inserting Eq. (3) into Eqs. (14) and (15) gives

$$\Pr[s_{it}-\mathbf{C}_\mathbf{i}\mathbf{r}_t+q_{it}-l_{it}-d_{it}\geqslant s_{i,t+1,\min}]\geqslant(1-\alpha) \tag{17}$$

$$\Pr[s_{it}-\mathbf{C}_\mathbf{i}\mathbf{r}_t+q_{it}-l_{it}-d_{it}\geqslant s_{i,t+1,\max}]\leqslant(1-\beta) \tag{18}$$

where $\mathbf{C}_i$ represents row i of the routing matrix and α, β=acceptable *risks* of failure to satisfy the constraints. These expressions can be rearranged as

$$\Pr[q_{it}\leqslant s_{i,t+1,\min}-s_{it}+\mathbf{C}_\mathbf{i}\mathbf{r}_t+l_{it}+d_{it}]\leqslant\alpha \tag{19}$$

$$\Pr[q_{it}\leqslant s_{i,t+1,\max}-s_{it}+\mathbf{C}_\mathbf{i}\mathbf{r}_t+l_{it}+d_{it}]\geqslant(1-\beta) \tag{20}$$

and can now be expressed in the following deterministically equivalent form:

$$\mathbf{C}_\mathbf{i}\mathbf{r}_t\leqslant s_{it}-s_{i,t+1,\min}-l_{it}-d_{it}+F_{it}^{-1}(\alpha) \tag{21}$$

$$\mathbf{C}_\mathbf{i}\mathbf{r}_t\geqslant s_{it}-s_{i,t+1,\min}-l_{it}-d_{it}+F_{it}^{-1}(1-\beta) \tag{22}$$

These constraints essentially *tighten* the restrictions on reservoir releases as the desired *risk* levels α, β decrease, thereby encouraging more conservative operational strategies. However, these risk constraints are *conditioned* on the current storage levels $\mathbf{s}_t$, which are also random variables. ReVelle et al. (1969) attempted to remove this dependency by restricting release policies to a simple *linear decision rule* (LDR)

$$r_{it}=s_{it}-b_{it} \tag{23}$$

where optimization is now performed on the parameters b_{it}. Several extensions to the LDR have been developed (e.g., Houck et al. 1980; Halliburton and Sirisena 1984; Stedinger 1984), but its extreme simplicity appears to be incompatible with complex reservoir system operations.

Loucks and Dorfman (1975) showed that chance constrained models are overly conservative and generate operational rules that exceed the prescribed reliability levels. This means that α, β do not represent the true risk associated with violating storage constraints, but can only be regarded as parameters that influence risk aversion in the solution. True risk must be estimated by performing Monte Carlo analyses on the proposed operational policies. Simonovic and Marino (1982) developed a reliability programming (RP) model that assigns economic loss estimates to the risk parameters and incorporates them as decision variables in the optimization. Difficulties in creating these economic loss estimates has limited the use of RP methods.

Stochastic Linear Programming Models

The deterministic LP formulation of the reservoir system optimization problem of Eqs. (1)–(6) assumes that all future inflows and other random phenomena are known with certainty. A more realistic assumption is that first period decisions can be made with certainty, but future decisions and their consequences are random. The so-called *two-stage* problem is formulated to minimize total costs (or maximize net benefits) from first stage decisions, plus the total expected costs (or net benefits) of future decisions, which depend on the first stage decisions and future random inflow realizations (Kall and Wallace 1995). If several scenarios of future streamflow time series have been generated, each with an assumed probability of occurrence, then a *deterministic equivalent* problem can be formulated for each possible inflow sequence (or scenario). Future reservoir release decisions are specified that would be made as a consequence of the occurrence of each scenario. Only the first stage decisions are actually implemented, since future decisions are not known with certainty. Following implementation of the first stage decisions, the problem is reformulated starting with the next period decisions and solved over the remainder of the operational horizon.

The difficulty with this formulation is that an ample number of possible scenarios results in an extremely large-scale linear programming problem. This can be reduced through application of Benders decomposition which projects the original large-scale problem onto the coupling variables, solves the resulting smaller subproblems via a dual formulation, and then solves a *master problem* which coordinates the subproblem solutions until an overall optimum is found for the original problem. Jacobs et al. (1995) applied stochastic linear programming using Benders decomposition to the Pacific Gas and Electricity hydropower system in northern California. Generalized network flow optimization is applied to the multireservoir system, with nonlinearities in the power calculations modeled using piecewise linear approximations. Decomposition of the large-scale linear programming problem into many smaller network flow optimization problems results in significant computational savings over attempts at direct solution. Seifi and Hipel (2001) applied two stage stochastic linear programming with recourse to the Great Lakes Reservoir System using an interior point method for solving the resulting large-scale LP.

The explicit stochastic linear programming model originally proposed by Thomas and Watermeyer (1962) discretizes storage and releases into given discrete levels, with discrete probabilities of occurrence of those levels stipulated as the decision variables in this formulation. Expected benefits of reservoir operations are maximized, with ''optimal'' probabilities of release levels conditioned on current discrete storage and inflow levels. Unfortunately, this formulation produces what are termed ''randomized release rules'' represented by probabilities of a particular release decision being made rather than actual release guidelines. Also, the combinations of discrete joint probabilities that must be calculated results in an extremely large-scale linear programming model, particularly when applied to multireservoir systems. Detailed discussion of this approach and its application to multireservoir systems can be found in Loucks et al. (1981).

Extensions of nonlinear programming to the stochastic case for multireservoir systems are rare due to the intense computational requirements. Ahmed and Lansey (2001) proposed a method based on the parameter iteration method of Gal (1979) involving quadratic approximation of future benefits and parameterization of operating policies for hydropower systems. Computational requirements are alleviated through a Lagrangian decomposition procedure, but the authors fail to mention the likelihood of existence of *duality gaps* in this formulation due to the nonconvexity of the objective function.

Stochastic Dynamic Programming Models

Stochastic dynamic programming (SDP) models attempt to solve the following DP recursion relation adapted to stochastic problems

$$F_t(\mathbf{s}_t) = \max_{\mathbf{r}_t} \ (\text{or} \ \min) \ \mathbf{E}_{\mathbf{q}_t}[\alpha_t f_t(\mathbf{s}_t, \mathbf{r}_t, \mathbf{q}_t) + F_{t+1}(\mathbf{s}_{t+1})] \quad (24)$$

Often referred to as a Markov decision process, this formulation assumes that unregulated inflows are temporally uncorrelated, although spatial correlation may be included. Extensions to lag-1 models requires specification of previous period inflows as additional state variables:

$$F_t(\mathbf{s}_t, \mathbf{q}_{t-1}) = \max_{\mathbf{r}_t} \ (\text{or} \ \min) \ \mathbf{E}_{\mathbf{q}_t|\mathbf{q}_{t-1}} \ [\alpha_t f_t(\mathbf{s}_t, \mathbf{r}_t, \mathbf{q}_t) + F_{t+1}(\mathbf{s}_{t+1}, \mathbf{q}_t)] \quad (25)$$

Simplified decision rules need not be assumed, and only probability distributions are used for deriving optimal policies with no presumption of foreknowledge of future inflow events. Steady-state feedback control policies $\mathbf{r}_t^*(\mathbf{s}_t, \mathbf{q}_{t-1})$ are generated which allow reservoir system operators to incorporate hydrologic uncertainty into reservoir release decisions. These policies are calculated by the value iteration or policy iteration methods (Howard 1960). The latter method is referred to as successive approximations by Dreyfus and Law (1977), whereby seasonal cycles of the SDP model are solved until optimal policies become stationary for each season t (e.g., calendar month).

The so-called inverted form of the SDP formulation

$$F_t(\mathbf{s}_t, \mathbf{q}_{t-1}) = \max_{\mathbf{s}_{t+1}} \ (\text{or} \ \min) \ \mathbf{E}_{\mathbf{q}_t|\mathbf{q}_{t-1}} \ [\alpha_t f_t(\mathbf{s}_t, \mathbf{r}_t, \mathbf{q}_t) + F_{t+1}(\mathbf{s}_{t+1}, \mathbf{q}_t)] \quad (26)$$

provides optimal storage guidecurves $\mathbf{s}_{t+1}^*(\mathbf{s}_t, \mathbf{q}_{t-1})$. Labadie (1993a) developed optimal storage guide curves for operation of Valdesia Reservoir in the Dominican Republic. Application of the guidecurves produced significant improvements over historical operations, using the same information that would have been available to reservoir operators during the historical period. Several other researchers have successfully applied SDP to single reservoir problems, such as Stedinger et al. (1984); Huang et al. (1991); and Vasiliadis and Karamouz (1994). Unfortunately, extensions of SDP to multireservoir systems are more aggravated by state dimensionality than in the deterministic case, particularly when spatial correlation of unregulated inflows must be maintained. One of the few multireservoir applications of SDP was conducted by Tejada-Guibert et al. (1995) on the Trinity-Shasta Reservoir system of California.

The sampling stochastic dynamic programming approach of Kelman et al. (1990) employs a scenario-based method similar to stochastic linear programming, but using DP as the solution algorithm. This method overcomes the complexities of representing multireservoir operations as a Markov decision process and accounting for all spatial and temporal dependencies in the stochastic process. Unfortunately, the method fails to alleviate the dimensionality problems associated with SDP, and is yet to be applied to multistate, multireservoir systems. As with all scenario-based approaches, questions arise as to the extremely small joint probabilities of occurrence of specific scenarios, particularly over extended time horizons.

The methods of IDP, DPSA, and DDP have been useful techniques for solving multireservoir DP problems in the deterministic case. Attempts to extend these methods to stochastic problems have not in general been successful, mainly since these methods are highly dependent on knowledge of the system state vector $\mathbf{s}_t$ with certainty. Sherkat et al. (1985) attempted to extend DPSA to the stochastic case by successive adjustment of stationary release policies one reservoir at a time. Unfortunately, the release policies generated for a particular reservoir are dependent only on storage and inflows to that reservoir, thereby ignoring important spatial dependencies in the reservoir system. Ponnambalam and Adams (1996) attempt to overcome this disadvantage by using two or three state variables at each iteration, with one of the state variables representing the aggregate storage potential of reservoirs not yet considered. This allows optimal stationary release policies to include spatial correlations to at least some extent. A somewhat similar approach is proposed by Archibald et al. (1997) whereby a sequence of three-dimensional SDP problems are solved, with states representing the current reservoir, aggregate states of upstream reservoirs, and an approximation of the downstream reservoirs. Braga et al. (1991) applied an approach similar to that of Sherkat et al. (1985) to the multireservoir system of the Companhia Energetica de Sao Paulo, Brazil, but attempted to account for spatial correlation of inflows. Unfortunately, this method is incapable of generating general operating rules since it determines specific reservoir release decisions assuming current storage levels and previous month inflows are known. In addition, only transition probabilities for the current month are considered in this formulation, with optimal benefits of future operations calculated deterministically.

Trezos and Yeh (1989) derived an extension of DDP for stochastic multireservoir problems, but Ouarda (1991) observed that the method converged to suboptimal solutions. Ouarda (1991) found the primary deficiency of the method to be the lack of calculation of optimal feedback decision rules. El-Awar et al. (1998) modified the algorithm of Trezos and Yeh (1989) to include calculation of optimal multilag feedback policies of the form:

$$\mathbf{r}_t^*(\mathbf{s}_t,\mathbf{q}_{t-1},\mathbf{q}_{t-2})=\overline{\mathbf{A}}_t\mathbf{s}_t+\overline{\mathbf{B}}_t\mathbf{q}_{t-1}+\overline{\mathbf{C}}_t\mathbf{q}_{t-2}+\overline{\mathbf{d}}_t \qquad (27)$$

Instead of directly calculating optimal release $\mathbf{r}_t^*$, coefficients in the matrices $\overline{\mathbf{A}}_t$, $\overline{\mathbf{B}}_t$, $\overline{\mathbf{C}}_t$, and vector $\overline{\mathbf{d}}_t$ are optimized. This structure allows incorporation of multilag hydrologic information into the reservoir system operating policies, with full inclusion of spatial dependencies. Although a nominal optimal state trajectory $\mathbf{s}_t^*$ is calculated, optimal policies retaining variability of the system state are calculated as quadratic expansions around these nominal state trajectories. El-Awar et al. (1998) show that Eq. (27) is easily modified to include nonlinear operating rule structures.

Hall (1970) originally proposed a method of surmounting the dimensionality problem of DP for multireservoir systems by aggregating all reservoirs into an *equivalent reservoir*. Optimal policies for the aggregated reservoir are then decomposed into individual policies for each reservoir as constrained by the aggregate solution. Turgeon (1980) extended this concept to large-scale hydropower systems using SDP. Instead of using reservoir storage as the state variable, a potential energy term is created for treating nonlinearities in the power calculations. Valdes et al. (1992) applied this technique to the four-reservoir lower Caroni hydropower system in Venezuela. Disaggregation was performed not only spatially, but temporally, resulting in daily operational policies from the monthly equivalent reservoir policies.

The state aggregation approach reached an advanced state with Saad et al. (1996) by incorporating neural networks as a means of improving the disaggregation process to account for nonlinear dependencies between the system elements. The method was successfully applied to finding long-term operational policies for Hydro-Quebec's five-reservoir hydropower system on the La Grande River. The difficulty with state aggregation/decomposition methods is the loss of information that occurs during the aggregation process.

Stochastic Optimal Control Models

Stochastic optimal control theory extends OCT to a solution of problems in the presence of uncertainty or *noise*. The so-called discrete-time linear quadratic Gaussian control (LQG) problem is one where Eq. (1) is quadratic, the system dynamics equations [Eq. (3)] are linear with the inflows represented as independent Gaussian error terms, and all other constraints [i.e., Eqs. (4)–(6)] are relaxed. Under these conditions, optimal linear feedback decision rules of the form

$$\mathbf{r}_t^*(\mathbf{s}_t)=\mathbf{K}_t\mathbf{s}_t+\mathbf{c}_t \qquad (28)$$

can be derived from the matrix Riccati equations (Bryson and Ho 1975) or continuous dynamic programming (Dreyfus and Law 1977). Determination of $\mathbf{K}_t$, $\mathbf{c}_t$ is accomplished by efficient recursive calculations that begin with the final period and proceed backwards in time. The *certainty equivalence principle* (Bryson and Ho 1975) states that although the feedback control law of Eq. (28) is derived by replacing the Gaussian error terms (i.e., hydrologic inflows) with their means, it remains optimal even in the presence of random errors. More realistic formulations model streamflows as AR(n) or *ARIMA* processes.

The most serious weaknesses of the LQG model when applied to multireservoir systems are the restriction to quadratic objective functions and relaxation of the control and state-space constraints [i.e., Eqs. (4)–(6). Ouarda and Labadie (2001) proposed an optimal control formulation using the aforementioned OCT algorithm subject to chance constraints similar to Eqs. (21) and (22) with assigned risk parameters α, β. This deterministic equivalent formulation results in optimal open loop release and storage trajectories $\mathbf{r}_t^*$, $\mathbf{s}_{t+1}^*$ ($t=1,...,T$) that are not directly useful for implementation in the stochastic case. However, they may be valuable for developing quadratic approximations of the nonquadratic objective function expanded around these trajectories, and for linearizing any nonlinear terms in Eq. (3). Optimal linear feedback decision rules [Eq. (28)] can now be developed for this approximate problem. Although certainly suboptimal, these policies may be useful for stochastic control of complex, large-scale reservoir system optimization problems that would defy solution by other methods. Shim et al. (1994) report on successful application of this approach to the five-reservoir system in the Han River Basin, South Korea.

Multiobjective Optimization Models

The primary objective function of Eq. (1) can be concisely represented as $f(\mathbf{s},\mathbf{r})$ where the vectors $\mathbf{s}$, $\mathbf{r}$ represent reservoir storage and releases for each site over all time periods. The problem of Eqs. (1)–(6) is now

$$\underset{\mathbf{s},\mathbf{r}}{\text{maximize}} \quad f(\mathbf{s},\mathbf{r}) \qquad (29)$$

subject to Eqs. (3)–(5), and

$$\overline{f}_j(\mathbf{s},\mathbf{r})\geqslant\varepsilon_j \quad \text{for } j=1,...,m \qquad (30)$$

where the latter constraints can be regarded as additional objectives that are treated parametrically. The *epsilon-constraint method* adjusts the ε_j targets to develop nondominated solutions defining a Pareto optimal surface of tradeoffs between the objectives (Goicoechea et al. 1982). These optimal solutions for each parametric set of ε_j values may be obtained using any of the aforementioned optimization algorithms deemed most appropriate for the particular system in either an ISO or ESO structure. Yeh and Becker (1982) applied the epsilon constraint method to multiobjective analysis of the Central Valley Project in California, considering tradeoffs between hydropower generation and water supply objectives.

Alternatively, the *weighting method* commensurates all of these objectives into a scalar value by assigning subjective weights (or relative magnitudes of importance) to each objective

$$\underset{\mathbf{s},\mathbf{r}}{\text{maximize}} \quad f(\mathbf{s},\mathbf{r})+\sum_{j=1}^{m} w_j\overline{f}_j(\mathbf{s},\mathbf{r}) \qquad (31)$$

varying the relative weights w_j also produces a nondominated solution set for evaluation of tradeoffs. Ko et al. (1992) compared these two methods for multiobjective evaluation of the Han River Reservoir system in Korea. The four objectives evaluated were: (1) maximizing total energy production; (2) maximizing firm energy; (3) maximizing minimum downstream discharges for water supply and water quality maintenance purposes; and (4) maximizing the reliability of satisfying downstream water supply requirements. The latter objective was evaluated using chance constraints as an ESO problem. It was concluded that the epsilon constraint method is more efficient since the weighting method may develop nonunique solutions for differing sets of weights. For large numbers of objectives, however, the latter method may be preferable.

Selecting the most preferred solution may be accomplished using goal programming (Loganathan and Bhattacharya 1990) or compromise programming (Zeleny 1982). Eschenbach et al. (2001) report that preemptive goal programming is employed within the *RiverWare* decision support system and applied to TVA's power and reservoir system. Preemptive goal programming involves setting goals for a primary objective and temporarily ignoring all other objectives. If this results in numerous nonunique solutions, then goal attainment for a secondary objective is optimized over this set of nonunique solutions, and so on. This method only works well if massive nonuniqueness of solutions is attained at each level. Otherwise, secondary objectives will receive inadequate consideration in the multiobjective analysis. In all of these cases, application of ISO is difficult because of the heavy computational load required for obtaining multiobjective solutions. These methods are usually applied deterministically for operational planning purposes. If a finite number of discrete solutions are selected from the Pareto optimal set, they can be further ranked using the methods of multicriteria decision analysis (MCDA) such as *ELECTRE* (Goicoechea et al. 1982), the analytical hierarchy process (AHP) (Saaty 1980), discrete compromise programming (Goicoechea et al. 1982), or *PROMOTHEE* (Brans et al. 1986).

Real-Time Control with Forecasting

Several of the aforementioned optimization models have been adapted for use in real-time control of reservoir systems. Implicit and explicit stochastic optimization methods can be applied to determining long-range guidecurves and policies over weekly, monthly, or seasonal time increments. Real-time optimal control models are then designed to *track* these long-term guidelines over shorter time horizons in hourly (or less) or daily time increments. For this case, flow routing and scheduling of individual hydropower units is often important, as well as real-time forecasting of inflows and demands (i.e., both water and power). Several authors have examined the importance of forecasting in real-time control of reservoir systems, such as Labadie et al. (1981); Mishalani and Palmer (1988); and Georgakakos (1989b). These studies conclude that use of forecasting is preferable, even in the presence of errors, to *reactive* control that ignores forecasts.

Wasimi and Kitanidis (1983) applied the discrete-time LQG model for optimal daily flood control operations in the multireservoir system of the Des Moines River basin, Iowa. The system state-space is expanded to include basin rainfall-runoff relationships and Muskingum-type streamflow routing. In this way, measured rainfall is the input to the model, and the forecast lead-time is based on natural time lagging and attenuation from the rainfall-runoff and routing processes. Although a fully dynamic closed-loop solution is attained over all future time steps, only the current time step open-loop solution is actually implemented. A Kalman filter updates estimates of observed system states, and the LQG algorithm is again solved beginning with the next time step. Bertsekas (1987) has termed this as an *open-loop feedback* process. The quadratic objective function is designed to penalize deviations from given ideal system states and releases, which must be determined from long-term operational planning studies.

The combined control-estimation model developed by Loaiciga and Marino (1985) uses a state-space formulation similar to Wasimi and Kitanidis (1983), but with inflows modeled as an AR(1) process and uncertainties in system measurements included. McLaughlin and Velasco (1990) extend this approach to complex hydropower systems in monthly time increments and apply it to the two-reservoir system in the Caroni River Basin of Venezuela. These models are essentially unconstrained optimization procedures and are not applicable to real-time control problems with binding constraints on reservoir storages and releases. McLaughlin and Velasco (1990) propose a heuristic procedure whereby the unconstrained optimal solution is simply truncated to feasible values prior to implementation. Philbrick and Kitanidis (1999) point out that the accuracy of these *certainty equivalence* based methods degrades for reservoir systems that increasingly deviate from the assumptions associated with the LQG model. Although they demonstrate the superiority of stochastic dynamic programming (SDP) in these cases, they offer no suggestions on how to overcome the severe computational requirements of SDP when applied to multireservoir systems.

Georgakakos and Marks (1987) proposed an extended LQG (ELQG) algorithm allowing inclusion of binding constraints on system state and release variables. Georgakakos (1989a) applied ELQG to the three-reservoir hydropower system on the Savannah River, Georgia. Constraints on releases are explicitly maintained, with chance constraints similar to Eqs. (24) and (25) employed on system storage levels. The latter are incorporated into the objective function using penalty terms (or barrier functions) similar to the OCT algorithm. Instead of unconstrained solutions, a series of constrained quadratic approximate solutions are obtained similar to a feasible SQP algorithm with simple bound constraints. A *reduced* approximate quadratic problem is projected onto the space of the decision variables (i.e., reservoir releases) similar to the GRG method, which removes direct consideration of the state-space constraints. The latter are accounted for by successive solutions with increasing penalty terms in the objective function. A key element of ELQG is the expansion of the system state representation to include both mean and covariance estimates on reservoir storage. Although a highly complex algorithm, ELQG is clearly superior to the previous methods for real-time control of reservoir systems where binding constraints exist on the state and decision variables. Georgakakos et al. (1997) further applied ELQG to hydropower scheduling in the Lanier-Allatoona-Carters system in Georgia, and included optimal scheduling and loading of turbine units.

Several studies have focused on the importance of incorporating realistic flow routing techniques in real-time control of reservoir systems, especially during flood control operations. Unver and Mays (1990) explicitly incorporate the linearized St. Venant equations for fully dynamic, unsteady flow routing into a real-time optimal control model for reservoir systems. The GRG algorithm is applied directly to reservoir gate controls as the primary decision variable, although an augmented Lagrangian function is used to deal with the state-space constraints. The primary difficulty of this algorithm is the need to calculate complex Jacobian matrices defined from partial derivatives of the St. Venant equations. The optimal control model is incorporated into a real-time flood management system for the Highland Lakes system in the Lower Colorado River Basin of Texas. Forecasted inflows are based on rainfall measurements input into a watershed rainfall-runoff model, similar to the approach of Wasimi and Kitanidis (1983).

Labadie (1993b) avoids difficulties in calculating derivatives for the St. Venant equations by using sets of *routing coefficients* in the state equations [i.e., Eq. (3)] that are iteratively updated through successive solution of the OCT algorithm with a fully dynamic unsteady flow routing model. Excellent convergence characteristics are observed when the combined algorithm is applied to real-time control of in-system detention storage of stormwater runoff for a portion of the Seattle combined sewer system. Since decision time steps are in 10–15 min intervals, inflow predictions from an urban rainfall-runoff model are further extrapolated using an ARMA-type forecasting model. An open-loop feedback procedure is employed, although forecasted inflows are treated deterministically. Shim et al. (2002) applied the routing coefficient method for optimal real-time flood control operations in the Han River Basin, Korea. A geographic information system processes spatial rainfall data in real-time for input into an artificial neural network (ANN) for inflow forecasting. The DPSA algorithm provides an efficient optimization procedure for generating optimal operating policies for the multireservoir system which are updated hourly.

Hayes et al. (1998) expanded the state-space in reservoir system optimization to directly include dynamic routing of water quality constituents through impoundments and stream reaches. Eq. (3) is augmented to include state dynamic equations on temperature and dissolved oxygen. The OCT algorithm is applied to develop optimal daily reservoir release schedules that maximize hydropower revenues, subject to constraints on water quality maintenance. Application to daily scheduling in the Cumberland River basin reservoir system in Tennessee indicates that significant improvement in downstream water quality conditions can be achieved with only modest losses in hydropower benefits.

Howard (1994) shows how optimization models can be incorporated into decision support systems for real-time reservoir control through linkage with supervisory control and data acquisition systems (SCADA), as well as real-time hydrologic and power load forecasting models. With monitoring and telemetry equipment now relatively inexpensive, a real-time decision support system can support all data management functions, provide real-time hydrologic and power load forecasts, generate effective displays of current system status, and allow operators to both simulate impacts of proposed operational controls and actually execute those controls from the interface.

Heuristic Programming Models

All of the foregoing optimization models are algorithmic procedures, meaning that well-structured, convergent solution processes are applied to quantitative information. In contrast, heuristic programming methods are based on rules-of-thumb, experience, or various analogies applied to both quantitative and qualitative information. Unlike most of the optimization algorithms, heuristic programs cannot guarantee termination to even local optimal solutions. These methods strive for acceptable or *satisfying* solutions, but they are often capable of achieving global optimal solutions to problems where traditional algorithmic methods would fail to converge or *get stuck* in local optima.

Genetic algorithms (GA) are categorized under the general heading of evolutionary programming (EP) in that they perform optimization through a process analogous to ‘‘the mechanics of natural selection and natural genetics’’ in the biological sciences (Goldberg 1989). Three heuristic processes of reproduction, crossover, and mutation are applied probabilistically to discrete decision variables that are coded into binary strings. Rather than generating progressions of single solutions, as with all of the preceding optimization algorithms, a GA produces groups or *populations* of solutions whose *offspring* display increasing levels of *fitness* (i.e., objective function values). Michalewicz (1996) showed that GAs representing decision variables with floating point or real number coding are more computationally efficient than binary coded GAs for problems requiring accurate real number calculations.

A disadvantage of GAs is the difficulty of explicitly accounting for constraints (particularly inequality constraints) and maintaining feasible solutions in the population. Constraints are generally indirectly accounted for through the use of penalty terms incorporated into the fitness (or objective) function, although Michalewicz (1996) describes evolution strategies that allow explicit consideration of constraints. Unfortunately, these methods are generally problem specific and must be modified for each new application. An example is an application of a GA by Ilich (2001) to a reservoir system optimization problem where the ‘‘iterative scheme was built directly into the solver in order to ensure that each generated solution is feasible.’’ Although Ilich (2001) claims this method to be a ‘‘replacement for standard LP solvers used in basin allocation models,’’ it means that each application requires recoding and fine tuning of the solver, which diminishes the value of this procedure as a generalized optimization tool.

Otero et al. (1995) applied a GA to determining minimum stormwater detention storage capacities and optimal operating rules for managing freshwater runoff into the St. Lucie Estuary along the southeast coast of Florida. Generalized, piecewise linear reservoir operating rules are used, with the GA directly manipulating the breakpoint locations on the rule curves for each season. The GA is linked with a daily hydrologic simulation model operating over a 27 year historical period and performs frequency analysis on mean monthly inflows resulting from the current operating rules. These are compared with the ideal frequency distributions using appropriate goodness-of-fit criteria. The objective function also includes penalty terms that attempt to minimize required storage capacities, as well as discourage violation of various operational constraints.

The significant advantage of the GA is that it can be directly linked with hydrologic and water quality simulation models without requiring simplifying assumptions in the model or calculation of derivatives. The GA adjusts *populations* of release rule structures based on predictions of the impacts of the rules as provided by the simulation model. Extensive frequency analyses can be conducted during the system simulation, resulting in discrete probability distributions and various risk measures that can be directly included in the objective function. Measures of system resilience (i.e., rate of recovery after occurrence of failure) and vulnerability (i.e., severity of consequences of failure) (Hashimoto et al. 1982), which are difficult to explicitly include in algorithmic procedures, are easily incorporated into a GA-simulation model linkage.

The key to successful application of the GA in the Otero et al. (1995) study is the optimization of parameters representing operating rule structures, rather than actual period-of-record releases over each time step. Oliveira and Loucks (1997) propose a similar approach, which is applied to defining multiple reservoir operating policies using system rule curves and individual storage target balancing functions. Sharif and Wardlaw (2000) propose application of a GA to direct optimization of period-of-record releases as an alternative to deterministic optimization approaches such as DDDP. However, the advantage of a GA lies not in its computational efficiency, but rather the robust ability to solve highly nonlinear, nonconvex problems. The expensive computational requirements of a GA make it ill-suited for ISO or ESO applied to multireservoir systems unless operating policies can be parameterized in some way.

Cai et al. (2001) describe an application of GAs to solving large-scale nonlinear water management problems over multiple periods, such as for ISO. The GA only optimizes over a limited number of complicating or *coupling* variables such that when fixed, allow decomposition of the original problem into many small linear programming problems. Bouchart and Hampartzoumian (1999) applied a GA to identify appropriate inflow sequences for training of a reinforcement learning (RL) model. Reinforcement learning provides strategies for solving problems similar to large-scale stochastic dynamic programming problems without the need for explicit knowledge of the state transition probability function (Kaelbling et al. 1996).

Although not classified as an optimization technique per se, artificial neural networks (ANN) may be useful as an alternative to multiple regression analysis for determining optimal rules from ISO. An ANN is a ''computational paradigm inspired by the parallelism of the brain.'' Artificial neurons or nodes are simple processing units that produce outputs as nonlinear functions of weighted sums of the inputs to that node. The ANN is particularly valuable in performing classification and pattern recognition functions for processes governed by complex nonlinear interrelationships. Raman and Chandramouli (1996) used an ANN for inferring optimal release rules conditioned on initial storage, inflows, and demands. Results of a deterministic DP model for the Aliyar reservoir in Tamil Nadu, India for 20 years of bimonthly data serve as a *training set* for the ANN. The training of an ANN is an optimization process, usually by a gradient-type *back-propagation* procedure, which determines the values of the weights on all interconnections that best explain the input-output relationship. Chandramouli and Raman (2001) extended this approach to developing operating rules for multireservoir systems.

Raman and Chandramouli (1996) claim that simulation of rules obtained from the trained ANN outperforms rules produced by linear regression analysis, as well as optimal feedback laws obtained from explicit stochastic optimization using SDP. Other uses of ANN may be in representing the DP optimal return or cost-to-go function $F_t(\mathbf{s}_t, \mathbf{q}_{t-1})$ with fewer sampling points, thereby creating the potential for solving high dimensional stochastic dynamic programming problems for reservoir system optimization. This is the basis for neurodynamic programming, as proposed by Bertsekas and Tsitsiklis (1996).

An alternative approach to inferring operating rules from historical operations or ISO of reservoir systems is through use of fuzzy rule-based (FRB) modeling. Fuzzy sets provide a nonfrequentist approach to dealing with uncertainty and vagueness that are not bound by the laws of probability measure theory. Fuzzy sets provide a means of translating linguistic descriptors into a usable numerical form. Fuzzy sets define degrees of truth of membership in a set by means of fuzzy membership functions.

Shrestha et al. (1996) propose that inputs to reservoir operating policies (e.g., initial storage, inflows, and demands), as well as outputs (e.g., historical release policies or results from ISO) can be described by fuzzy relations. *Degrees of fulfillment* of these fuzzy inputs are combined to produce fuzzy output relations which can be *defuzzified* to produce a *crisp output* (e.g., reservoir release decision). Similar to an ANN, results of ISO of a reservoir system produce a *training set* which the fuzzy rule-based system attempts to analyze using various methods such as the weighted counting algorithm or least-squares methods for adjusting the fuzzy numbers. Shrestha et al. (1996) report excellent results in using an FRB system to replicate historical operations for Tenkiller Lake, Okl. It is likely that the FRB approach could be extended to multireservoir systems for inferring operating rules from training sets produced by ISO.

Fuzzy sets have also been integrated into optimization algorithms as a means of representing vagueness and uncertainty in system characteristics and objectives. Fontane et al. (1997) used linguistically described reservoir objectives from surveys of decision makers to develop fuzzy membership functions on diverse objectives such as water supply, flood control, and recreation. These were incorporated into an implicit stochastic dynamic programming model for evaluating degrees of satisfaction and expectations of success in achieving these objectives. Luhandjula and Gupta (1996) proposed the integration of fuzzy sets into ESO models as a means of appropriately treating uncertainty in complex systems.

Conclusions

There are a few areas of application of optimization models with a richer or more diverse history than in reservoir system optimization. Although opportunities for real-world applications are enormous, actual implementations remain limited or have not been sustained. Shepherd and Ortolano (1996) report on personal communications with system operators stating that they ''don't like being told what to do...'' or a preference to make decisions ''in his own way.'' Many examples of the lack of success in implementation of reservoir system optimization models occur in public works agencies with vague performance objectives. Often, in these cases, the avoidance of difficulties or perceived system failure are the dominant goals, rather than improving efficiency or reducing costs. This is not necessarily true for many private or quasi-public water and power systems where strong financial and revenue-based incentives exist for deployment of optimization methods. Opportunities for implementation of reservoir system optimization models may grow as the public demands greater performance-based accountability in water management agencies. Reservoir system operators may increasingly rely on sophisticated computer modeling tools to better respond to new environmental and ecological constraints for which they have little experience to draw on.

This writer is convinced that the keys to success in implementation of reservoir system optimization models are: (1) improving the levels of trust by more interactive involvement of decision makers in system development; (2) better ''packaging'' of these systems, as suggested by Goulter (1992); and (3) improved linkage with simulation models which operators more readily accept. For the latter, increased application of heuristic programming methods is particularly important, which many system analysts have been slow to adopt because they lack a strong scientific or theoretical foundation. The ability of genetic algorithms to be linked directly with trusted simulation models is a great advantage. In addition, past difficulties in inferring operating policies from implicit stochastic optimization models may be alleviated through applications of fuzzy rule-based systems and neural networks. The computational challenges of explicit stochastic optimization may also be overcome through judicious application of these heuristic techniques.

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