
Propagation of drought through groundwater—a new approach using linear reservoir theory

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Abstract:

The effect of drought on groundwater heads and discharge is often complex and poorly understood. Therefore the propagation of a drought from groundwater recharge to discharge and the influence of aquifer characteristics on the propagation was analysed by tracking a drought in recharge through a linear reservoir. The recharge was defined as a sinusoid function with a period of 1 year. The decrease in recharge owing to drought was simulated by multiplying the recharge during 1 year with a drought fraction between 0 and 1, which represents a decrease in the recharge of 100 to 0%, respectively. The droughts were identified using the threshold level approach, with a threshold that is constant in time. For this case analytical formulations were derived, which express the drought duration and deficit in the groundwater discharge in terms of the decrease in recharge, the reservoir coefficient that characterizes aquifer properties and the height of the threshold level. The results showed that the delay in the groundwater system caused a shift of the main part of the decrease in recharge from the high-flow to the low-flow period. This resulted in an increase in drought deficit for discharge compared with the drought deficit for recharge. Also the development of multiyear droughts caused an increase in drought deficit. The attenuation in the groundwater system caused a decrease in drought deficit. In most cases the net effect of these processes was an increase of drought deficit as a result of the propagation through groundwater. Only for small droughts the deficit decreased from recharge to discharge. The amount of increase or decrease depends on the reservoir coefficient and the severity of the drought. Under most conditions a maximum in the drought deficit occurred for a reservoir coefficient of around 200 days. Copyright © 2003 John Wiley & Sons, Ltd.

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INTRODUCTION

Natural droughts are recurring phenomena, which affect all components of the water cycle (Wilhite, 2000). When a drought affects groundwater, it is called a groundwater drought. Natural droughts can be classified into meteorological, agricultural and hydrological droughts, where hydrological droughts include both streamflow and groundwater droughts (Hisdal *et al.*, 2001). Like the other types of natural drought, groundwater droughts are caused by low precipitation possibly in combination with high evapotranspiration. Groundwater droughts develop only slowly from meteorological droughts. A deficit in precipitation (meteorological drought) can result in a recharge deficit, which in turn causes lowered groundwater heads and a deficit in groundwater discharge (Changnon, 1987; McNab and Carl, 1991; van Lanen and Peters, 2000). Another cause of groundwater drought is abstraction, which may enhance naturally occurring droughts, but in case of overexploitation may create groundwater droughts (Acreman *et al.*, 2000; Custodio, 2000; Foster, 2000; van Lanen and Peters, 2000). The consequences of groundwater drought are diverse. The direct effects are lower groundwater heads and a decrease of the groundwater flow to riparian areas, springs and streams. For shallow groundwater, capillary rise to the vegetation will decrease, which may affect wetlands and crop yield

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negatively. Also well yields may decrease and shallow wells may even dry up (Calow *et al.*, 1999). One of the most irreversible effects of prolonged groundwater drought is the slow intrusion of salt water.

To date, little research has been devoted to the occurrence and propagation of drought in groundwater. Yet, from regression analyses relating drought in stream flow to catchment properties, it is well known that geology is one of the main factors influencing hydrological drought (Zecharias and Brutsaert, 1988; Vogel and Kroll, 1992). Recently, however, the interest in groundwater drought has increased (Gottschalk *et al.*, 1997; Robins *et al.*, 1997; White *et al.*, 1999), specifically in connection with climate change (Leonard, 1999). Recent research includes work from Price *et al.* (2000), who propose outflow from storage in the unsaturated zone as the source of larger-than-expected groundwater discharge from chalk aquifers during drought. Marani *et al.*, (2001) analysed the influence of geomorphological controls on groundwater discharge and in particular the influence on the behaviour of floods and droughts. A recent attempt to analyse the propagation of droughts from recharge to groundwater discharge by Peters *et al.*, (2001) revealed several problems. A main difficulty is the lack of understanding of the way aquifer characteristics influence the propagation of droughts through groundwater. Therefore this study aims to investigate systematically how droughts are propagated from recharge to groundwater heads and discharge, and to evaluate how this propagation depends on aquifer characteristics. To keep the analysis transparent, a synthetic recharge function was defined and the groundwater system was simulated as a linear reservoir with a reservoir coefficient representing the aquifer characteristics. These choices enabled the derivation of analytical expressions, which express the drought duration and deficit in terms of the decrease in recharge and the reservoir coefficient.

DEFINITION OF DROUGHT EVENTS

Although most people have quite a strong notion about what a drought is, no precise common definition of drought exists. Therefore it is important to start with a description of how droughts will be defined in this paper. Central in most definitions of drought is the concept of a water deficit over a limited period of time (up to several years), but extended in space (Dracup *et al.*, 1980; Beran and Rodier, 1985; Wilhite and Glantz, 1985; McNab and Carl, 1991; Hisdal *et al.*, 2001; NDMC, 2002). In this paper the term 'drought' is used to describe events that are selected from a time-series using the threshold level approach, which was first described by Yevyevich (1967) (Figure 1). This definition has three major consequences:

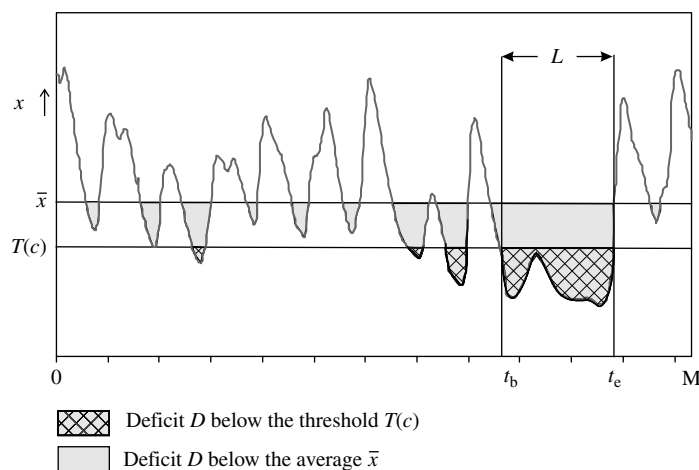


Figure 1. Illustration of the threshold level approach and the definition of the threshold level $T(c)$ for a time-series of variable x with average \bar{x} of length M containing four droughts, where c is the drought criterion, L is drought duration and t_b and t_e are start and end time of the drought

1. droughts can occur only during periods that have a low water availability in the absolute sense and not during periods that are dry only relative to the normal situation;
2. permanent low water availability is not called drought but 'aridity';
3. not only extreme events are called droughts but all events that have a low availability of water—these are often called 'minor droughts' or 'non-extreme events' (Hisdal *et al.*, 2002).

This definition of droughts is close to the meaning of the expression 'low flow', which is often used for stream flow (Smakhtin, 2001).

The threshold level approach is based on the theory of runs or crossing theory, which studies the statistical properties of runs above and below a given threshold level (Bras and Rodriguez-Iturbe, 1985). The threshold level approach is also called the peak over threshold (POT) approach and the series of events below the threshold are called the partial duration series (PDS).

For any hydrologically relevant variable x (e.g. recharge, discharge, heads or storage) the deficit D or run sum below the threshold for a particular drought is calculated as

$$D = \int_{t_b}^{t_e} (T - x(t)) dt \quad (1)$$

where t_b and t_e are the start and end date of the drought and T is the threshold (Figure 1). Please note that $x(t_b) = x(t_e) = T$ and that $x(t) < T$ for $t_b < t < t_e$. The duration L is calculated as

$$L = t_e - t_b \quad (2)$$

An important step is the determination of an appropriate threshold level T . The threshold can be a naturally occurring threshold, for example, the streamflow rate below which ships can no longer navigate a river or the level below which no groundwater can be extracted from a specific well. However, in many cases, no natural threshold is available and therefore the threshold is derived from observations. The method most commonly used is the percentile approach, which means that a percentile (e.g. 70 or 80 percentile) from the cumulative frequency distribution is chosen as the threshold. However using the threshold level approach for data that contain many zeros or which are highly skewed (e.g. recharge or discharge from intermittent streams) is problematic. Solutions to this problem that are presented in literature use a very low percentile (e.g. 20 percentile by Santos and Gonçalves-Henriques, (1999)), only part of the data (e.g. only winter) or annual data, and are not suitable for this study because they all would require using different event definitions in one analysis. This would make comparisons between drought deficits very difficult. Therefore a new approach to derive threshold levels is introduced, which is based on relating the total deficit below the threshold to the total deficit below the average. The total deficit below a threshold is the sum of the deficits of all droughts below this threshold or in other words the sum of the PDS. This is illustrated in Figure 1. Thus the threshold function $T(c)$ can be defined as follows

$$\int_0^M (T(c) - x(t))_+ dt = c \int_0^M (\bar{x} - x(t))_+ dt \quad (3)$$

where

$$\begin{aligned} x_+ &= x & \text{if } x \geq 0 \\ &= 0 & \text{if } x < 0 \end{aligned}$$

M is the length of the time series and c is the *drought criterion*, which determines the value of the threshold level. The drought criterion c is the ratio of the deficit below the threshold to the deficit below the average. If $c = 1$ the threshold level is equal to the average \bar{x} , where \bar{x} is the average of variable x , which is calculated as

$\bar{x} = \frac{1}{M} \int_0^M x(t) dt$. If $c = 0$ the threshold level is equal to the minimum of x . This definition of the threshold also ensures that the total drought deficit decreases with decreasing amplitude of $x(t)$, something which is not necessarily true when the threshold is determined with percentiles.

THE PROPAGATION OF A DROUGHT IN A LINEAR RESERVOIR

An overview of the approach used to analyse the propagation of droughts from recharge to discharge is presented in Figure 2. A description of the recharge and the derivation of the discharge will be presented later. The groundwater system or aquifer was approximated by a linear reservoir. The discharge rate from the linear reservoir is given by

$$q = \frac{1}{j} S \quad (4)$$

where q is the discharge rate ($L T^{-1}$), S is the storage of the reservoir (L) and j is the reservoir coefficient (T). According to non-linear drainage theory (Kraijenhof van de Leur, 1962; Rorabaugh, 1964; Ritzema, 1994), the reservoir coefficient j can be interpreted, in specific cases, as $j = \mu l^2 / \pi^2 kD$, where μ is the storage coefficient (–), l is the distance between streams (L) and kD is the transmissivity ($L^2 T^{-1}$). This interpretation is valid for horizontal flow in an isotropic medium between parallel drains. For naturally drained aquifers, a more

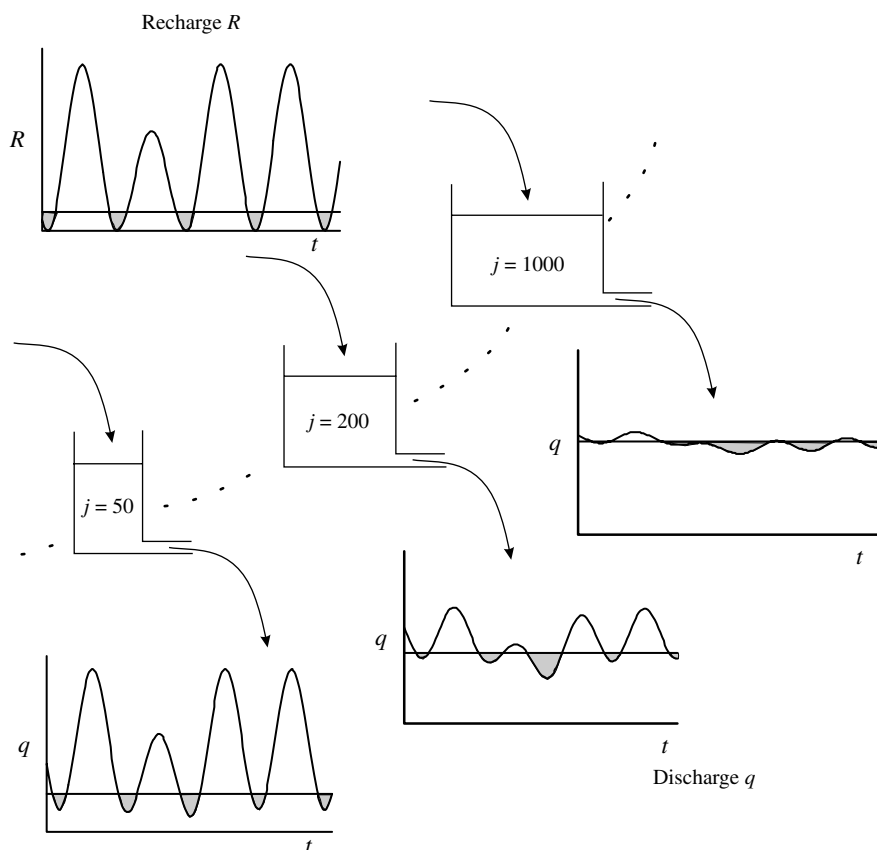


Figure 2. Schematic overview of the procedure followed to analyse the propagation of drought through groundwater. The sinusoid recharge function serves as input to the linear reservoir, which is characterized by the reservoir coefficient j (in days)

general interpretation is the response time ($\mu l^2/kD$) (Birtles and Wilkinson, 1975). Values of j of 1 to 20 or 30 days are generally applicable to artificially drained fields and values of 300 to 2000 days to discharge from aquifers (VVL, 1992).

Combining Equation (4) with the equation for the conservation of mass results in the following ordinary, first-order, non-homogeneous differential equation

$$\frac{dq(t)}{dt} = \frac{1}{j}(R(t) - q(t)) \quad (5)$$

where $R(t)$ is the recharge rate ($L T^{-1}$). As groundwater is approximated by a linear reservoir, the principle of superposition can be used to analyse the propagation of a drought through the reservoir. Let the recharge rate $R(t)$ be composed of two parts, namely the normal recharge rate $R_n(t)$ and the decrease in recharge rate owing to drought $R_d(t)$ then

$$R(t) = R_n(t) - R_d(t) \quad (6)$$

From the principle of superposition it follows that discharge rate also can be written as

$$q(t) = q_n(t) - q_d(t) \quad (7)$$

where $q_n(t)$ is the normal discharge and $q_d(t)$ is the decrease in discharge as a result of the decrease in recharge. The total decrease in recharge DC_R and discharge DC_q is

$$DC_R = \int_{-\infty}^{\infty} R_d(t) dt \quad (8)$$

As groundwater is simulated as a linear reservoir, the transient discharge from the reservoir and the storage in the reservoir (represented by the head) are identical, except for a scaling factor and dead storage. From now on only the discharge will be analysed, but the analysis would have been identical for the reservoir storage. For more complicated reservoirs, such as natural aquifers, the relationship between storage and discharge may not be trivial and a separate analysis may be required.

RECHARGE AND DISCHARGE

Definition of the recharge

The next step is to define the recharge function in Equation (6). This recharge function should be simple enough to allow the derivation of an analytical expression, but also should be sufficiently realistic to allow a meaningful analysis. The recharge is described by a sinusoidal shape, which represents annually recurring recharge (Figure 2). The normal recharge rate is defined as follows

$$R_n(t) = R_0(1 + \sin(2\pi\omega t)) \quad (9)$$

where R_0 ($L T^{-1}$) is the long-term average recharge rate and ω (T^{-1}) is the frequency of the recharge. The dry period is defined as a single year with decreased recharge. The decrease in the recharge is defined as

$$R_d(t) = \begin{cases} (1 - f_d)R_n(t) & \text{for } \frac{3}{4\omega} \leq t \leq \frac{7}{4\omega} \\ 0 & \text{for } t < \frac{3}{4\omega} \text{ and } t > \frac{7}{4\omega} \end{cases} \quad (10)$$

where f_d ($-$) is the *drought fraction*, which determines the amount of decrease in the recharge. For all examples in this article, ω is $1/365 \text{ day}^{-1}$ and R_0 is $0.685 \text{ mm day}^{-1}$ (250 mm year^{-1}), an amount of recharge that is common in subhumid climates such as the UK or The Netherlands. The drought fraction specifies the ratio between the recharge in the drought year and the recharge in an average year. For example, for $f_d = 0.6$

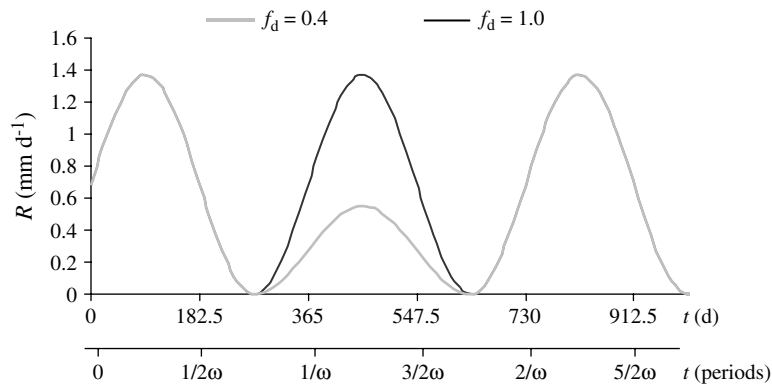


Figure 3. Recharge functions R for $f_d = 1.0$ and $f_d = 0.4$ for the time expressed both in days and periods of the sine function (ω is the frequency)

the $R(t)$ in the drought year is $0.6R_n(t)$. An example of the recharge for $f_d = 1$ (no drought) and $f_d = 0.4$ is given in Figure 3. The sinusoidal function assumes a systematic sequence of periods with low recharge (summer or dry season) and high recharge (winter or wet season). The defined recharge function in combination with the event definition with a threshold that is constant in time, implies that a drought will occur each year.

The selected definition of recharge limits the analysis in two ways. First, this type of recharge with a clearly seasonal recharge peak each year, mainly occurs in humid or subhumid climates. For arid or semi-arid climates the recharge would be much more erratic. Secondly, the analysis is limited to droughts that originate in the wet period, as mainly the recharge during the peak (wet season) is decreased (Figure 3). However, it is expected that this type of drought is especially important for groundwater (Warren, 1994; Seely, 1999).

Derivation of the discharge

From Equations (9) and (10) and Equations (5) and (7) the discharge rate can be derived. The normal discharge rate $q_n(t)$ is

$$q_n(t) = R_0 \left(1 + \frac{1}{jA} \sin(2\pi\omega t + B) \right) \tag{11}$$

The decrease in discharge rate $q_d(t)$ owing to a decrease in the recharge rate is

$$q_d(t) = \begin{cases} 0 & \text{for } t < \frac{3}{4\omega} \\ (1 - f_d)q_n(t) - R_0 \left(1 - \frac{1}{(jA)^2} \right) (1 - f_d)e^{-\frac{t}{j} + \frac{3}{4\omega j}} & \text{for } \frac{3}{4\omega} \leq t \leq \frac{7}{4\omega} \\ R_0 \left(1 - \frac{1}{(jA)^2} \right) (1 - f_d)e^{-\frac{t}{j} + \frac{3}{4\omega j}} (e^{\frac{1}{\omega j}} - 1) & \text{for } t > \frac{7}{4\omega} \end{cases} \tag{12}$$

where

$$A = \sqrt{(1/j)^2 + (2\pi\omega)^2}$$

$$B = \arcsin \left(\frac{-2\pi\omega}{A} \right)$$

These equations can easily be verified by inserting $R(t)$ and $q(t)$ in Equation (5). The discharge $q(t)$ is illustrated in Figure 2 for $f_d = 0.6$, $c = 0.1$ and three values of the reservoir coefficient ($j = 50$, $j = 200$ and $j = 1000$ days). The decrease in discharge $q_d(t)$ is presented in Figure 4 for $f_d = 0.6$. Please note that the term jA also can be written as $\sqrt{1 + (2\pi\omega j)^2}$. This term combines the effect of the response time

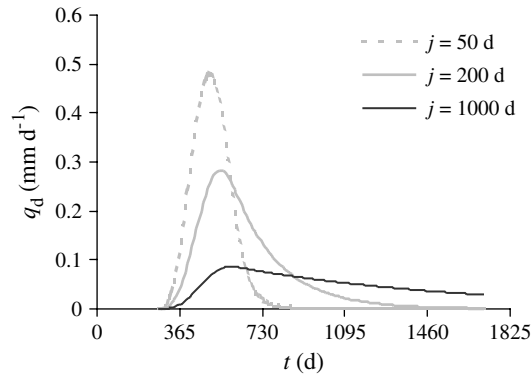


Figure 4. Decrease in discharge $q_d(t)$ as a result of a decrease in the recharge ($f_d = 0.6$)

of the groundwater reservoir defined by the reservoir coefficient with the frequency of the recharge. From $q_n(t)$ (Equation 11) it can be seen that the transition through the linear reservoir results in an amplitude magnification (attenuation) of $1/jA$ and a phase shift (delay) of B . This is illustrated well in Figure 4. For increasing reservoir coefficients the maximum in the decrease comes later (delay) and is less high (attenuation), the right tail becomes longer and the duration increases. Of course, the total amount of decrease is identical irrespective of the reservoir coefficient. In the following we will examine how the attenuation and delay interact with the drought event definition to form the drought duration and deficit.

DERIVATION OF DROUGHT DURATION AND DEFICIT

Threshold level

In this section expressions describing the duration and deficit of the drought will be derived. First, however, an expression for the threshold level will be derived according to the event definition defined previously (Equation 3). The threshold is derived from the normal situation, i.e. for $R_n(t)$ and $q_n(t)$. In Appendix A the derivation of the thresholds for the recharge T_R and the discharge T_q are described. The thresholds are

$$T_R(c) = R_0 + R_0X(c) \quad (13)$$

$$T_q(c) = R_0 + \frac{R_0X(c)}{jA} \quad (14)$$

where $X(c)$ is a shape function that depends only on the drought criterion c and which transforms the drought criterion based on the deficit (Equation (3), Figure 1) to a threshold level. Variable $X(c)$ is presented as a function of the drought criterion c in Figure 5.

Duration and deficit

Recharge. With the expression for the threshold level (Equation 13), the duration and deficit of the droughts in the recharge (recharge droughts) can be calculated. In Figure 6 this is illustrated for two possible situations. For $f_d = 0.6$ the reduction in recharge is not very large and the peak at 456 days ($5/4\omega$) still exceeds the threshold. Two droughts result with identical duration and deficit ($Dw1_R$ and $Dw2_R$). Both droughts last less than 1 year (within-year drought). However, when the peak in the recharge remains lower than the threshold (e.g. for $f_d = 0.15$), one drought with a duration of more than 1 year results (multiyear drought, here 2-year drought with deficit Dm_R). This has to be taken into account when calculating the deficit and duration of the droughts. The duration L_R and deficit D_R of the recharge droughts are determined according to Equations (1)

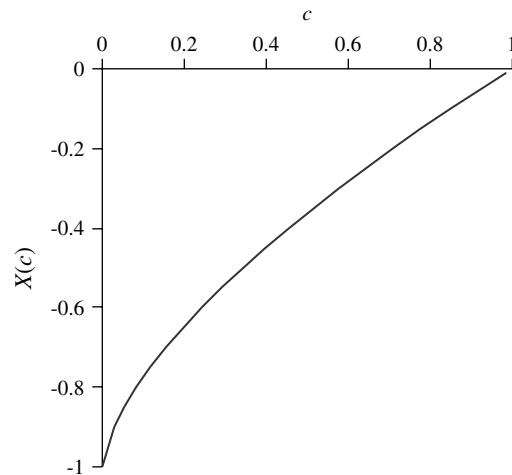


Figure 5. Shape function $X(c)$ as a function of drought criterion c , for the threshold in the recharge and discharge (Equations 13 and 14)

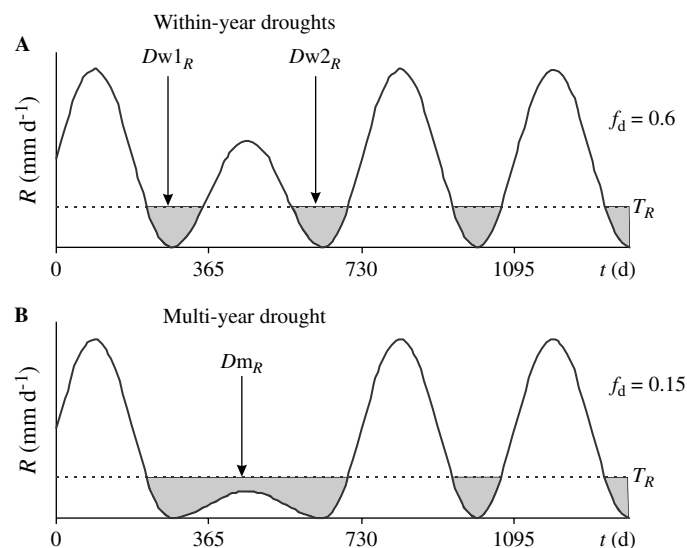


Figure 6. Recharge drought below threshold T_R ($c = 0.3$) for drought fraction $f_d = 0.6$ (A) and for $f_d = 0.15$ (B). $Dw1_R$ and $Dw2_R$ are the drought deficit for the first and second within-year drought, Dm_R is the drought deficit for the multiyear drought (2-year here) (Equations 1 and 19). Please note: the additions w (for within-year drought) and m (for multiyear drought) have been added to distinguish between within-year and multiyear droughts

and (2). When determining the times of intersection between the recharge function and the constant T_R , we have to consider in which part of the recharge function the intersections will be, namely whether it is in the drought part ($3/4\omega \leq t < 7/4\omega$ days) or not. For the 2-year drought both times of intersection are in the normal part R_n of the recharge. For the two within-year droughts one of the intersections will be in the normal part and the other one in the drought part (Figure 6).

The times of intersection t_e and t_b for the first within-year drought and the 2-year drought are

$$t_b = \frac{1}{2\omega} - \frac{1}{2\pi\omega} \arcsin(X(c)) \quad (15)$$

$$t_e = \begin{cases} \frac{1}{\omega} + \frac{\arcsin X_{R,d}(c)}{2\pi\omega} & \text{for the first within-year drought} \\ \frac{1}{\omega} + \frac{\arcsin X(c)}{2\pi\omega} & \text{for 2-year drought} \end{cases} \quad (16)$$

where

$$X_{R,d}(c) = \frac{T_R(c) - f_d R_0}{f_d R_0} = \frac{X(c) + 1 - f_d}{f_d} \quad (17)$$

For the second within-year drought, t_e and t_b are exchanged. The duration of the recharge droughts L_R is

$$L_R(c) = \begin{cases} \frac{1}{2\omega} + \frac{1}{2\pi\omega} [\arcsin X(c) + \arcsin (X_{R,d}(c))] & \text{for within-year droughts (Lw1}_R \text{ and Lw2}_R) \\ \frac{[3\pi + 2 \arcsin X(c)]}{2\pi\omega} & \text{for 2-year drought (Lm}_R) \end{cases} \quad (18)$$

Where $Lw1_R$ and $Lw2_R$ refer to the drought duration in the recharge of the first and second within-year drought and Lm_R to the deficit of the multiyear drought. Using Equations (15) and (16) for t_b and t_e , respectively, the following relationships are derived for the deficit of the recharge drought D_R

$$D_R(c) = \begin{cases} \frac{cR_0}{2\pi\omega} + \frac{f_d R_0}{2\pi\omega} \left[\sqrt{1 - (X_{R,d}(c))^2} + X_{R,d}(c) \arcsin (X_{R,d}(c)) + \frac{\pi}{2} X_{R,d}(c) \right] & \text{for within-year droughts (Dw1}_R \text{ and Dw2}_R) \\ \frac{cR_0}{\pi\omega} + \frac{f_d R_0}{\omega} X_{R,d}(c) & \text{for 2-year drought (Dm}_R) \end{cases} \quad (19)$$

Where $Dw1_R$ and $Dw2_R$ refer to the drought deficit in the recharge of the first and second within-year drought and Dm_R to the deficit of the multiyear drought (Figure 6).

Discharge. For the discharge the decrease is no longer limited to the period between $t = 3/4\omega$ and $t = 7/4\omega$, but it becomes longer, as was illustrated in Figure 4. This means that the times of intersection of the drought in the discharge (discharge drought) can no longer be determined analytically, as it is not known beforehand in which part of the discharge function the intersection will be (part 1, $t < 3/4\omega$; part 2, $3/4\omega < t < 7/4\omega$; part 3, $t > 7/4\omega$). The example in Figure 7 shows that the deficit is spread out over at least three separate droughts. The main drought is the 2-year drought lasting from approximately 300 to 860 days. For this example the start of the drought t_b is in part 2 of the discharge function and the end of the drought t_e is in

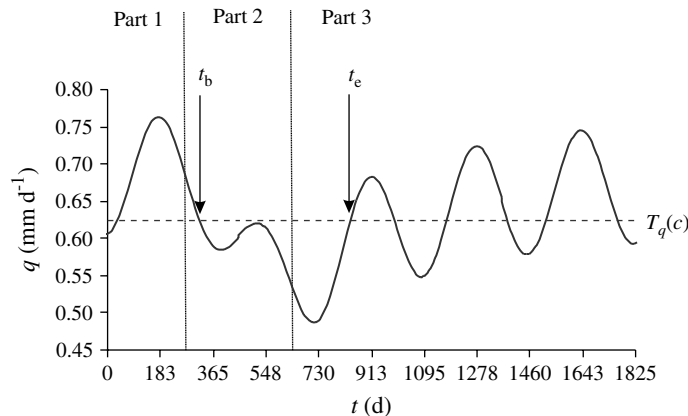


Figure 7. Drought derived from the discharge ($j = 500$ d). The times t_b and t_e denote the start and end of the drought respectively. Parts 1 to 3 denote the three parts of the discharge as defined in Equations (7), (11) and (12)

part 3 (Equation 12). For a somewhat smaller reservoir coefficient or a higher threshold, however, the start of the drought will be in part 1 of the discharge. As no analytical solution is possible, the times of intersection are determined numerically. With the numerical approximation of the times of intersection, duration L_q and deficit D_q of discharge droughts are determined according to Equations (2) and (1). The integral in Equation (1) is expanded analytically to arrive at an expression for D_q . Depending on the part the drought starts and ends in, different expressions result. An example of the expressions derived for D_q is presented in Appendix B.

RESULTS

Recharge

In Figure 8 the duration L_R and deficit D_R of the recharge droughts are presented as a function of the drought fraction f_d and the drought criterion c . The total decrease in recharge DC_R (Equation 8) is added for comparison in Figure 8B. The discontinuity in the lines is caused by the transition from one within-year to a 2-year drought. When the peak in the recharge during the drought no longer exceeds the threshold, the deficits $Dw1_R$ and $Dw2_R$ are summed to form one drought with deficit Dm_R (Figure 6). If the sum (duration or deficit) of the two within-year droughts would have been presented, the lines would have been continuous. Obviously, L_R increases with increasing drought criterion c and decreasing drought fraction f_d , with the major increase for the change from within-year droughts to two-year droughts (Figure 8A). For two-year droughts, L_R no longer depends on the drought fraction at all, because the start and end are only determined only by the normal recharge R_n . This is illustrated in Figure 6.

Generally the drought deficit D_R is much smaller than the actual decrease in recharge DC_R (Figure 8B). Only for very small droughts ($f_d \approx 1$) D_R is larger because D_R cannot become smaller than the deficit in a normal year ($f_d = 1$). For small droughts D_R is rather insensitive to the drought fraction f_d . For long droughts (multiyear droughts), on the other hand, D_R is linearly related to the drought fraction f_d and thus is parallel to DC_R . This is also evident from the equations for D_R (Equation (19), Dm_R).

Discharge

In Figure 9 an example of the results for the drought deficit D_q is presented from which the most important processes will be explained. Please note that the x -axis has a logarithmic scale. In Figure 9 three symbols are shown, which are labelled $Dw1_q$, $Dw2_q$ and Dm_q . These are the deficits for the first within-year drought

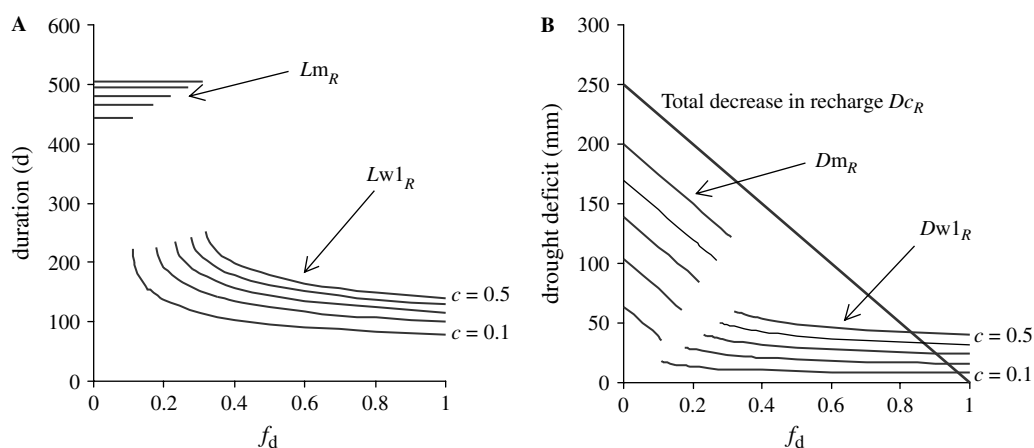


Figure 8. Recharge drought duration L_R (A) and deficit D_R (B) for five values of the drought criterion c : $c = 0.1$ to $c = 0.5$ (step 0.1). Also presented in figure B is the total decrease in recharge DC_R according to Equation (8)

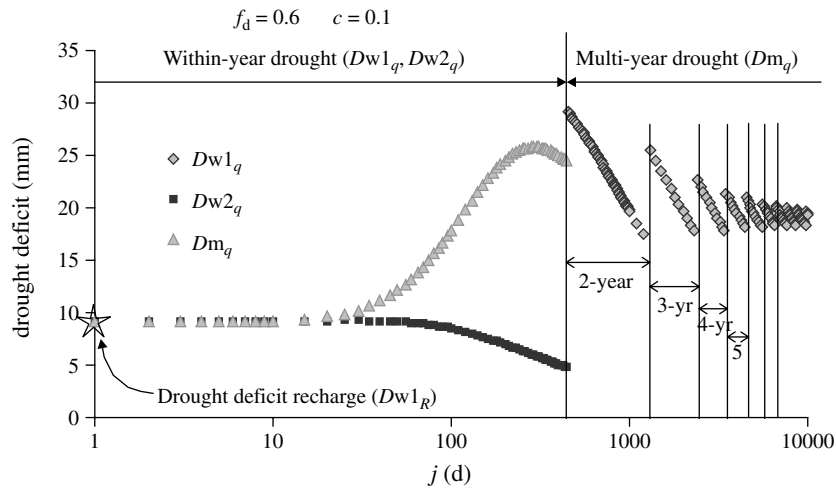


Figure 9. Deficit of the discharge droughts D_q as a function of the reservoir coefficient j for drought fraction $f_d = 0.6$ and drought criterion $c = 0.1$ for two within-year droughts ($Dw1_q$ and $Dw2_q$) and one multiyear drought (Dm_q) (Figure 6). Also presented is the deficit of the recharge drought $Dw1_R$ for $f_d = 0.6$ and $c = 0.1$

($Dw1$), for the second within-year drought ($Dw2$) and for the multiyear drought (Dm) (see also Figure 6). Also the deficit of the associated recharge drought D_R is indicated. The value of D_R is practically identical to the deficit of the discharge drought for reservoirs with a very small reservoir coefficient ($j \approx 1$), therefore D_R has not been indicated in the overview in Figure 11, which will be presented later. Overall we see a slight decrease in D_q for the first within-year drought ($Dw1_q$), a rise in D_q for the second within-year drought ($Dw2_q$) and a combination of sudden discontinuous increases ('jumps') and a decrease between the jumps for the multiyear droughts (Dm_q).

The decrease of $Dw1_q$ is caused by the increasing attenuation with increasing reservoir coefficient j (Figure 4). The increase of $Dw2_q$ is the result of the delay of the decrease in discharge $q_d(t)$ (Dc_q). The main part of Dc_q shifts from the wet to dry season with increasing j (Figure 4). For reservoir coefficients of up to $j = 290$ days the net effect of attenuation and delay is an increase in $Dw2_q$. For larger reservoir coefficients, the decrease in q owing to attenuation becomes larger than the increase caused by the delay and therefore D_q decreases. For multiyear droughts (Dm_q), every discontinuity ('jump') represents the addition of another year to the duration of the drought (Figures 7 and 9). The decrease in deficit in between the 'jumps' is again caused by the attenuation as explained earlier. The large number of discontinuities shows how long

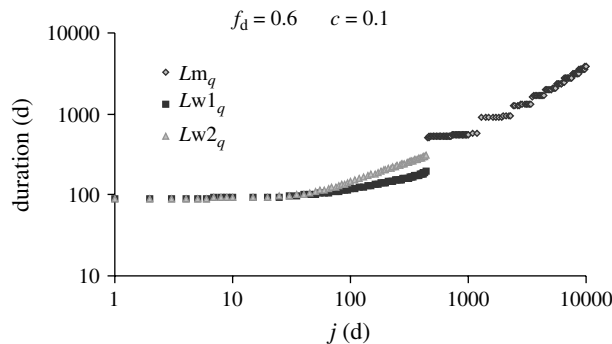


Figure 10. Duration of the discharge droughts L_q as a function of the reservoir coefficient j for drought fraction $f_d = 0.6$ and drought criterion $c = 0.1$ for two within-year droughts ($Lw1_q$ and $Lw2_q$) and one multiyear drought (Lm_q)

the droughts become for very large reservoir coefficients (> 10 years for $j > 10\,000$ day). These droughts are so long because the decrease in the recharge is followed by average recharge conditions (Figure 2) and this assumption becomes increasingly unlikely for longer droughts. In reality the droughts would have been ended by wetter than normal recharge conditions.

In Figure 10 the drought duration (L_q) for the example in Figure 9 is presented. This shows that the duration of the discharge droughts L_q only increases with increasing reservoir coefficient (Figure 10), which means

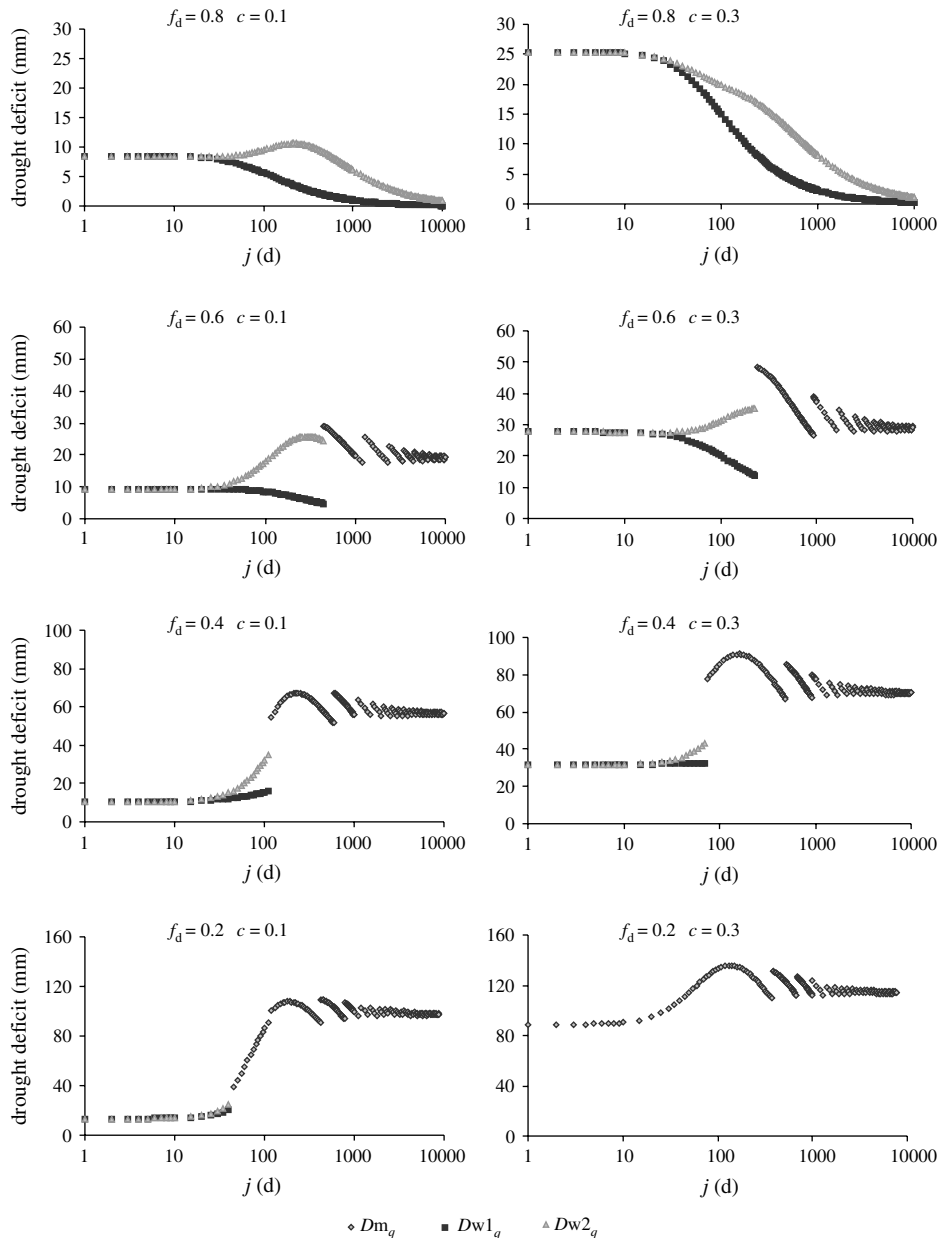


Figure 11. Deficit of the discharge drought D_q for several values of the drought fraction and drought criterion. For explanation of symbols see Figure 9

that discharge droughts last longer than recharge droughts. The duration of the second within-year drought is larger than that of the first within-year drought, because the second drought is affected more by the decrease in recharge. The major increase in duration ('jump') is caused by the transitions from within-year droughts to multiyear droughts.

In Figure 11 an overview is presented for discharge drought deficits with a decrease in recharge of 20% ($f_d = 0.8$) to 80% ($f_d = 0.2$) for two drought criteria: $c = 0.1$ and $c = 0.3$. The influence of the drought criterion will be discussed in the following section. Please recall that the deficit in the recharge is practically identical to the deficit for $j = 1$ (Figure 9). The general pattern as explained for the specific case with $f_d = 0.6$ and $c = 0.1$ (Figure 9) is evident in most examples. Only for $f_d = 0.8$ do no multiyear droughts develop and as a consequence the drought deficit becomes very low for large reservoir coefficients. The increase in drought deficit with increasing drought severity (decreasing drought fraction f_d) is much smaller for small reservoir coefficients (from 8.4 for $f_d = 0.8$ to 13.4 mm for $f_d = 0.2$ ($c = 0.1$)) than for large reservoir coefficients (from 0.8 for $f_d = 0.8$ to 98 mm for $f_d = 0.2$ ($c = 0.1$)). Obviously, multiyear droughts develop for smaller reservoir coefficients in the case of more severe droughts (f_d small) and higher thresholds (c large). For most of the examples presented in Figure 11 the deficit has a maximum. The average reservoir coefficient, where the maximum occurs, is $j = 205$ days (range: 140–290 days).

DISCUSSION

The results of the numerical experiment presented in Figures 9, 10 and 11 have been tested partially using real data by Peters and van Lanen (2003). Instead of a synthetic recharge signal, they used recharge based on observed meteorological data as input for a linear reservoir. Their results confirm the main results of this paper. Both in subhumid and semi-arid climate types a maximum in the discharge deficit occurs for intermediately large reservoir coefficients.

The results presented here also can be interpreted for a single reservoir or aquifer (for one value of j). A small decrease in the recharge translates into an even smaller drought in the discharge or no drought at all. A large decrease in the recharge translates into a large drought in the discharge.

The way in which the drought (decrease) in the recharge (R_d , Equations 6 and 10) is simulated influences the results. In this paper, the decrease in recharge was defined in the wet season and the decrease shifts to the dry season as a result of the delay of the reservoir. This causes the drought deficit to be larger in the discharge than in the recharge. Of course, it can easily be deduced that if the main decrease in recharge is in the dry season, then the delay would cause the decrease in discharge to shift in the direction of the wet season and this would result in a decrease in deficit with increasing reservoir coefficient. However, in most cases a decrease in recharge during the dry season is hardly possible owing to the physical processes determining the formation of recharge.

Influence of the drought event definition

Several aspects of the drought event definition influence the calculated drought duration and deficit. The influence of the height of the threshold level is straightforward (Figure 11): for a higher threshold level ($c = 0.3$) the drought duration and deficit are larger than for a lower threshold level ($c = 0.1$). For higher threshold levels, multiyear discharge droughts develop for less severe recharge droughts and for lower reservoir coefficients. A consequence of using a constant threshold as a drought event definition is the fact that the drought deficit is not always a good measure for the overall decrease in recharge or discharge (Figure 8). Because only the part below the threshold is analysed, deviations during normally high recharge are not taken into account. Thus, if the main interest is in volumes of water rather than water levels, this approach can be misleading. Therefore in some cases a different drought event definition is used, for example, for reservoir management (Montaseri and Adeloje, 1999).

The type of event definition has a more profound influence on the results than the height of the threshold level. If a variable threshold had been used instead of a threshold, which is constant in time, the results for the recharge would have been like the decrease in recharge DC_R in Figure 8B and the results for the discharge would have been like q_d (Figure 4). If a threshold had been chosen, which is constant in time and identical for all reservoir coefficients, the deficit volume would be largest for the recharge and for the groundwater discharge of the fastest responding groundwater systems (Figure 2). Instead of the threshold level approach, also the annual n -day minimum discharge, which is a commonly used measure for streamflow drought frequency analysis (Smakhtin, 2001), could have been used as a drought event definition. In Figure 12 the minimum discharge is presented as a function of the reservoir coefficient for different drought fractions. As expected, the minimum discharge increases with increasing reservoir coefficient and decreases with drought severity. The increase in minimum discharge with the reservoir coefficient is far from linear. In Figure 13 the decrease in minimum recharge compared with the minimum recharge for the normal discharge q_n is presented. This shows that for a reservoir coefficient of about 225 days the minimum discharge decreases most compared with the normal situation for the full range of drought fractions. This is close to the value of 205 days for which, on average, the maximum in the deficit occurs (Figure 11).

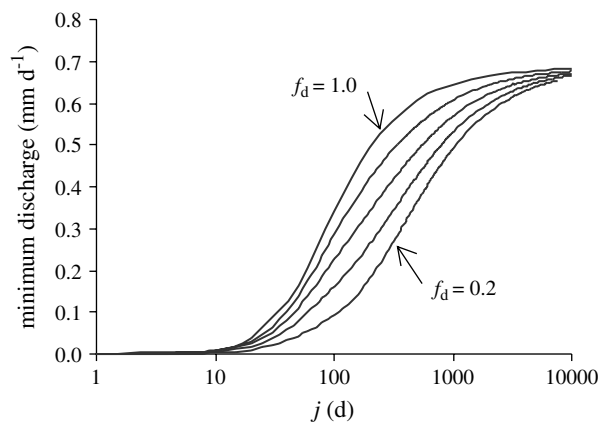


Figure 12. Minimum discharge for several values of the drought fraction: $f_d = 0.2$ to $f_d = 1.0$ (step 0.2) ($f_d = 1$ means no drought, normal recharge)

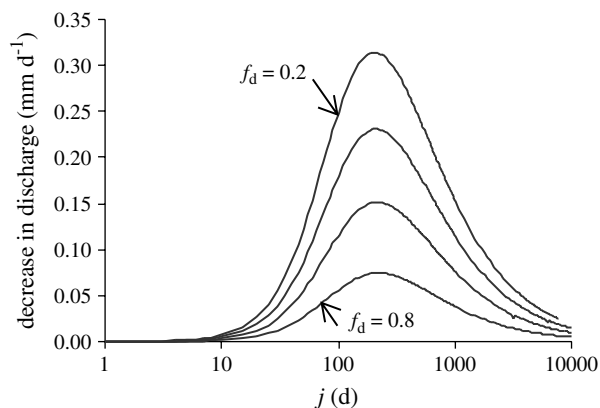


Figure 13. Decrease in minimum discharge with respect to the normal minimum discharge as a function of the reservoir coefficient, for several values of the drought fraction: $f_d = 0.2$ to $f_d = 0.8$ (step 0.2)

CONCLUSIONS

The simple approach that was used to simulate the propagation of droughts through a groundwater system was found to provide a clear overview of the effects of the propagation through a groundwater reservoir (namely attenuation and delay) on recharge drought. The approach, which consisted of a combination of a synthetic recharge function with a linear reservoir, enabled the analysis of a wide range of reservoir coefficients (aquifer characteristics) and drought severities. Although the effects of a reservoir (attenuation and delay) on inputs are well known, the effect of the propagation of a recharge drought (defined using a constant threshold) through a groundwater system is less straightforward and depends on the interaction of the attenuation and delay with the drought event definition. The analyses resulted in the following conclusions.

1. The *main effect of the attenuation* is a decrease of the deficit of discharge droughts compared with recharge droughts. The decrease in the drought deficit is caused by a decrease in the amplitude of the discharge compared with the recharge function. The drought duration remains practically constant or increases slightly.
2. The *main effect of the delay* is an increase in the duration and deficit of the discharge drought. As the main decrease in recharge was defined in the high flow period, the decrease in recharge shifted from the wet season in the direction of the following dry season.
3. A *combined, secondary effect* of the attenuation and delay is the development of multiyear droughts.

The net effect of these three effects on the duration and deficit of the groundwater discharge drought depends on the amount of decrease of the recharge and on aquifer characteristics represented by the reservoir coefficient. In most cases the deficit and duration of the discharge drought were larger than the deficit and duration of the recharge drought. Only for small droughts (20% decrease in recharge during 1 year) the deficit of the discharge drought was smaller. The amount of deficit increase from a recharge to a discharge drought depends on the reservoir coefficient, on the severity of the drought and on the height of the threshold level. In most cases, the largest increase in deficit occurred for groundwater systems with a reservoir coefficient of around 200 days. The increase in deficit was generally larger for more severe droughts and for lower threshold levels.

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APPENDIX A

Derivation of the threshold level

The threshold is derived from the normal situation, i.e. for $R_n(t)$ and $q_n(t)$. The derivation is based on the deficit and duration of one cycle ($1/\omega$ or 365 days). Based on the event definition (Equation 3), the following equation defines the threshold for the recharge (T_R)

$$\int_{t_b(T_R)}^{t_c(T_R)} (T_R - R(t)) dt = c \int_{t_b(R_0)}^{t_c(R_0)} (R_0 - R(t)) dt \quad (\text{A1})$$

Where t_b and t_c are the times of intersection between the threshold level and the recharge function (left-hand side) and the average recharge and the recharge function (right-hand side) respectively. For the threshold (T_R),

t_b and t_e enclose the period where $R(t) < T_R$. The times of intersection follow from equating the recharge $R(t)$ to the threshold

$$T_R = R_0(1 + \sin(2\pi\omega t)) \quad (\text{A2})$$

where T_R is the threshold level for the recharge. This results in

$$\begin{aligned} t_b &= \frac{1}{2\omega} - \frac{1}{2\pi\omega} (\arcsin(X_R)) \\ t_e &= \frac{1}{\omega} + \frac{1}{2\pi\omega} (\arcsin(X_R)) \end{aligned} \quad (\text{A3})$$

where

$$X_R = \frac{(T_R - R_0)}{R_0}$$

The times of intersection for the average are $1/2\omega$ and $1/\omega$. Substituting $R(t)$ in Equation (A1) results in

$$\int_{t_b}^{t_e} (T_R - R_0 - R_0 \sin(2\pi\omega t)) dt = c \int_{\frac{1}{2\omega}}^{\frac{1}{\omega}} (-R_0 \sin(2\pi\omega t)) dt \quad (\text{A4})$$

The left-hand side of Equation (A4) is (please note that: $\cos(\arcsin(x)) = \sqrt{1-x^2}$)

$$(T_R - R_0) \left(\frac{1}{2\omega} + \frac{1}{\pi\omega} (\arcsin(X_R)) \right) + \frac{R_0}{\pi\omega} \left(\sqrt{1 - X_R^2} \right) \quad (\text{A5})$$

The right-hand side of Equation (A4) is

$$\frac{cR_0}{\pi\omega} \quad (\text{A6})$$

Combining the solution of the right-hand side and left-hand side, results after rewriting in

$$\sqrt{1 - X_R^2} + X_R \arcsin X_R + X_R \frac{\pi}{2} = c \quad (\text{A7})$$

This means that the dimensionless number X_R , which was defined above, depends only on the drought criterion c in the drought event definition. The number X_R defined by Equation (A7) is from now on denoted as the function $X(c)$. The function $X(c)$ can be derived numerically. In Table AI some relevant values for $X(c)$ are listed. It is also possible to derive an expression for the threshold based on percentiles. Comparison of the two thresholds results in the following: $c = 0.1$ is equal to Q78, $c = 0.2$ is Q72, $c = 0.3$ is Q68, $c = 0.4$ is Q65 and $c = 0.5$ is Q62 (Q78 is the flow that is equalled or exceeded 78% of the time). Figure 5 gives $X(c)$ as a function of c . Thus for the threshold T_R the following equation holds

$$T_R(c) = R_0 + R_0X(c) \quad (\text{A8})$$

In the same way the threshold for the discharge (T_q) can be derived. This results in the following expression for the threshold for the discharge

$$T_q(c) = R_0 + \frac{R_0X(c)}{jA} \quad (\text{A9})$$

Table AI. Values of X as a function of c

c	$X(c)$
0.1	-0.778
0.2	-0.649
0.3	-0.541
0.4	-0.447
0.5	-0.360

APPENDIX B

Derivation of the deficit for the discharge

To arrive at analytical expressions for the deficit, several possibilities with regard to the timing of the intersection have to be taken into account. An overview of the different possibilities is presented below:

1. $t_b < 3/4\omega$ and $3/4\omega < t_e < 7/4\omega$
2. $t_b < 3/4\omega$ and $t_e > 7/4\omega$
3. $3/4\omega < t_b < 7/4\omega$ and $3/4\omega < t_e < 7/4\omega$
4. $3/4\omega < t_b < 7/4\omega$ and $t_e > 7/4\omega$

As an example the deficit for the first option is presented

$$D_q = \int_{t_b}^{\frac{3}{4}\omega} (T_q - q(t)) dt + \int_{\frac{3}{4}\omega}^{t_e} (T_q - q(t)) dt = D_{q,I} + D_{q,II} \quad (B1)$$

where

$$D_{q,I} = (T_q - R_0) \left(\frac{3}{4\omega} - t_b \right) + \frac{R_0}{2\pi\omega jA} \left[\cos\left(\frac{3}{2}\pi + B\right) - \cos(2\pi\omega t_b + B) \right] \quad (B2)$$

$$D_{q,II} = (T_q - f_d R_0) \left(t_e - \frac{3}{4\omega} \right) + jR_0 \left(1 - \frac{1}{(jA)^2} \right) (1 - f_d) e^{\frac{3}{4\omega j}} \left(e^{-\frac{t_e}{j}} - e^{-\frac{3}{4\omega j}} \right) \\ + \frac{f_d R_0}{2\pi\omega jA} \left[\cos(2\pi\omega t_e + B) - \cos\left(\frac{3}{2}\pi + B\right) \right] \quad (B3)$$

Please note that $\cos\left(\frac{3}{2}\pi + B\right) = \sin(B) = \frac{-2\pi\omega}{A}$.

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