Optimization of Large-Scale Hydropower System Operations

Mario T. L. Barros[1]; Frank T-C. Tsai[2]; Shu-li Yang[3]; Joao E. G. Lopes[4]; and William W-G. Yeh, Hon.M.ASCE[5]

Abstract: A practical monthly optimization model, called SISOPT, is developed for the management and operations of the Brazilian hydropower system. The system, one of the largest in the world, consists of 75 hydropower plants with an installed capacity of 69,375 MW, producing 92% of the nation's electrical power. The system size and nonlinearity pose a real challenge to the modelers. The basic model is formulated in nonlinear programming (NLP). The NLP model is the most general formulation and provides a foundation for analysis by other methods. The formulated NLP model was first linearized by two different linearization techniques and solved by linear programming (LP). A comparative analysis was made of the results obtained from the linearized and the NLP models. The results show that the simplest linearized model (referred to as the LP model) without iteration is suitable for planning purposes. For example, the LP model could be used in system capacity expansion studies or to explore various design parameters in connection with feasibility studies, where details in storage variation are not as important as the power production. With a good initial policy provided by the LP model, the successive linear programming (SLP) model produced excellent results with fast convergence. The NLP model, the most complex and accurate model in the suite, is particularly suited for setting up guidelines for real-time operations using inflow forecast with frequent updating. The performance of the NLP model was checked against the historical operational records, and the comparison yields indications of superior performance.

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Introduction

Brazil is replete with natural water resources and its hydropower system, one of the largest in the world, has an installed capacity of 69,375 MW producing 92% of the nation's electrical power. The system consists of 75 hydropower plants and a combination of storage reservoirs and run-of-river plants. The network covers the following eight basins in Brazil: (1) North (Amazon); (2) South Atlantic; (3) Tocantins; (4) Sao Francisco; (5) Southeast Atlantic; (6) Parana; (7) Uruguay; and (8) South Atlantic. Fig. 1 shows the spatial distribution of the hydropower plants. Five hydropower plants are responsible for producing 50% of the Natural Inflow Energy (NIE): (1) Itaipu Dam (14,000 MW); (2) Tucurui Dam (4,241 MW); (3) Xingo Dam (3,000 MW); (4) Paulo Afonso/Moxoto Dam (4,285 MW); and (5) Ilha Solteira/Tres Irmaos Dam (4,252 MW). If considering 75% of the NIE, the number of reservoirs responsible increases to only 17. This shows that the Brazilian hydropower system is composed of a few very large storage reservoirs and many medium- and small-sized hydropower plants. Presently, the system is operated in an integrated fashion. A firm established by the Brazilian Federal Government, the National System Operator (ONS), is in charge of the operations. ONS defines the monthly, weekly, and daily operational rules. The main operational objective is to maximize the potential energy of the system. The Brazilian energy commerce is in the process of changing. In 1998, ONS assigned the power contracts for the subsequent 5 years (1999–2003). A set of models (optimization and simulation) was used to define the energy that each reservoir must produce during this period. After the year 2003, the energy commerce in Brazil will be partially controlled by a free market. For the first time, excess energy (amount of energy over the contract level) will be available to the free market. In the future, the market will be completely free, but ONS intends to preserve the benefits of integrated operations. For a system such as the Brazilian one, it will be very important to have an optimization model to plan the operations in advance to support the management in their decision making. It is also important to point out that a small improvement in operations for a system of such size translates into enormous economic benefits.

Optimization techniques have become increasingly important over the last three decades in the management and operations of complex reservoir systems. Yeh (1985), Simonovic (1992), Wurbs (1993, 1996), ReVelle (1997), and Momoh et al. (1999a,b) have provided an extensive literature review and evaluation of various optimization methods and their corresponding models. The complexities of a multipurpose, multireservoir system generally require that release decisions be made by an optimization or simulation model. Most of the optimization models are based on some type of mathematical programming technique. In general, the available optimization methods include the following algorithms:

[1]Associate Professor, Dept. of Hydraulic and Sanitary Engineering, Univ. of Sao Paulo, Sao Paulo, Brazil. E-mail: mtbarros@usp.br

[2]Postdoctoral Fellow, Dept. of Civil and Environmental Engineering, UCLA, Los Angeles, CA 90095. E-mail: ftsai@seas.ucla.edu

[3]Formerly, Postdoctoral Fellow, Dept. of Civil and Environmental Engineering, UCLA, Los Angeles, CA 90095. E-mail: yang_shuli@hotmail.com

[4]Graduate Student, Dept. of Hydraulic and Sanitary Engineering, Univ. of Sao Paulo, Sao Paulo, Brazil. E-mail: jelopes@dglnet.com.br

[5]Professor and Chair, Dept. of Civil and Environmental Engineering, UCLA, Los Angeles, CA 90095. E-mail: williamy@seas.ucla.edu

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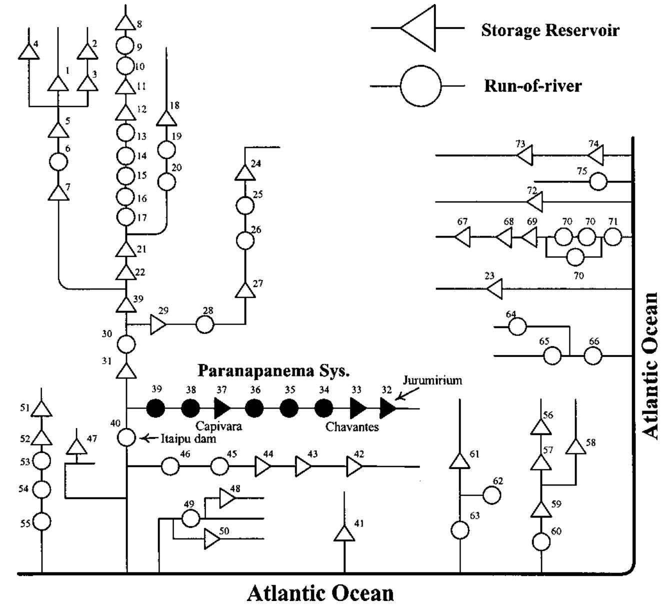

Fig. 1. Spatial distribution of hydropower plants

1. Linear programming (LP), including network flow models,
2. Quadratic programming (QP),
3. Dynamic programming (DP),
4. Nonlinear programming (NLP),
5. Mixed integer programming (MIP),
6. Interior point methods (IP), and
7. Nongradient-based search algorithms.

The choice of methods depends on the characteristics of the system being considered, on the availability of data, and on the objectives and constraints specified. Braga et al. (1998) developed a model for optimizing the Brazilian hydropower system based on the LP-DP method of Becker and Yeh (1974). Their model optimizes the operation month by month, and the LP-DP decomposition may lead to a near-sighted solution if the monthly ending storage condition is not properly specified. Barros et al. (2001) developed an NLP model to optimize the energy production of the Paranapanema subsystem, which consists of three storage reservoirs and five run-of-river plants (Fig. 1, Nodes 32–39). Their model considers the entire planning horizon and involves no decomposition. Preliminary results show that NLP is a viable approach. In the present paper, the basic NLP model developed by Barros et al. (2003) for the Paranapanema subsystem is extended to the entire Brazilian hydropower system of 75 hydropower plants. The system size, complexity, and the nonlinearities associated with hydropower generation pose a real challenge to the modelers. The new model, SISOPT, considers multiple objectives and optimizes a combination of objective functions. It consists of three optimization models—the LP, the NLP, and the SLP model.

A joint team of researchers from University of California, Los Angeles (UCLA) and the University of Sao Paulo (EPUSP), Brazil jointly developed the SISOPT model. The National Science Foundation (NSF) and its Brazilian counterpart Conselho Nacional de Desenvolvimento Cientifico e Technologico (CNPq) supported the collaborative research. During the development of the model, the team consulted with ONS, Brazilian Electric Energy Agency (ANEEL) and Brazilian Water Resources Agency (ANA).

Nonlinear Programming (NLP) Model

The basic model is formulated in terms of NLP, which offers the most general formulation and provides a foundation for analysis by other methods (Yeh 1985). With the drastic advancement in computing power and the development of effective nonlinear solvers in recent years, NLP has become a viable tool in solving large-scale water resources optimization problems (Peng and Buras 2000). The NLP model proposed herein is designed in such a way that it can be applied to many practical situations. Additionally, it can be easily incorporated into a decision support system that allows the user to update data, execute the model, and view the results graphically.

Traditionally, a reservoir system is designed for multiple purposes. There are several ways to solve a multiobjective optimization problem. Yeh and Becker (1982) used the constraint method to generate the trade-off curves among the five objectives considered for the California Central Valley Project. Can and Houck (1984) applied preemptive goal programming to the real-time operation of a multipurpose, multireservoir system. Their approach circumvents the need to assign penalty functions, but it does require that the system be highly redundant and that the solution be nonunique so that objectives can be satisfied sequentially. Loganathan and Bhattacharya (1990) outlined five goal-programming schemes and formulated the reservoir operation problem involving multiobjectives as a multiobjective linear program. Ko et al. (1992) made a comparative evaluation of several multiobjective optimization methods in connection with reservoir management and operations. They concluded that the constrained method (also referred to as the ϵ-constrained method) was most suitable for generating the tradeoffs among the competing objectives. Eschenbach et al. (2001) developed a multiobjective, preemptive linear goal-programming model for reservoir operation and applied it to the Tennessee Valley Authority network. Another way is to combine the various objectives by the weighting method or by treating some of the objectives as constraints.

For the Brazilian hydropower system, the proposed NLP model considers the following six objectives: (1) minimizing the loss of the stored potential energy; (2) minimizing storage deviations from targets; (3) maximizing total energy production; (4) minimizing spilled energy; (5) minimizing energy complementation; and (6) maximizing the profit derived from secondary energy. Note that certain combinations of the objectives are neither desirable nor feasible. We now quantify each of the six objectives considered in the NLP model.

1. Objective 1 minimizes the loss of the stored potential energy (Becker and Yeh 1974)

$$\min f_1 = \sum_t \sum_i (c_i' R_{i,t}' + c_i'' R_{i,t}'') \tag{1}$$

where $R_{i,t}'$=power release from reservoir i during time period t, in (m^3/s); $R_{i,t}''$=nonpower release from reservoir i during time period t (m^3/s); and c_i' and c_i'' are the weighting coefficients for power release and nonpower release (spill), respectively. To minimize the nonpower release, a large value of c_i'' is assigned.

2. Objective 2 minimizes the sum of the squares of storage deviations from targets, and can be represented by the following two different ways:

$$\min f_2 = \sum_t \sum_i (S_{i,t} - T_{i,t})^2 \tag{2a}$$

or

$$\min f_2 = \sum_t \left(\sum_i S_{i,t} - T_t \right)^2 \quad (2b)$$

where $S_{i,t}$=ending storage in the ith reservoir at the end of time period t, in Mm^3 (10^6 m^3); $T_{i,t}$=specified ending storage target at the ith reservoir at the end of time period t; and T_t=specified ending storage target for the summation of the ending storage vector of the entire system at the end of time period t.

3. Objective 3 maximizes total energy production

$$\max f_3 = \sum_t \sum_i (\xi_{i,t} R'_{i,t}) \quad (3)$$

where $\xi_{i,t}$=energy production function in MW/m^3/s per month.

4. Objective 4 minimizes total spilled energy

$$\min f_4 = \sum_t \sum_i (\xi_{i,t} R''_{i,t}) \quad (4)$$

5. Objective 5 minimizes energy complementation by the following quadratic function

$$\min f_5 = \sum_t \left[D_T - \sum_i (\xi_{i,t} R'_{i,t}) \right]^2 \quad (5)$$

where D_T=total energy demand, which includes not only hydropower but also other alternative sources of energy, such as the more expensive thermal or nuclear energy. The purpose of this objective is to minimize the use of alternative sources of energy, which, in turn, will minimize the total operational cost.

6. Objective 6 maximizes the profit derived from the secondary energy

$$\max f_6 = \sum_t p_t \left(\sum_i \xi_{i,t} R'_{i,t} - D_C \right) \quad (6)$$

where p_t=energy price during time period t (\$/MW); and D_C=contractual demand (system firm energy) during time period t (MW). In Brazil, the concept of firm energy is used to establish the energy contracts. Firm energy is defined as the average energy that can be produced on a long-term basis with a 5% risk. The amount of energy produced above the firm energy is called the secondary energy. The price of firm energy is higher than that of the secondary energy, but secondary energy can be sold in the open market.

We now formulate the composite objective function as follows:

$$\min Z = w_1 f_1 + w_2 f_2 - w_3 f_3 + w_4 f_4 + w_5 f_5 - w_6 f_6 \quad (7)$$

As we stated before, certain combinations of the objectives are neither desirable nor infeasible, and some of the objectives can be treated as hard constraints. The weighting coefficients ($w's$) reflect the priority of these objectives, and a judicious selection of their values is crucial to achieving a balance.

The constraint set includes the monthly energy demand, turbine capacity, maximum storage variation, flood control reservation, minimum storage, and minimum and maximum allowable releases through the turbine. Specifically, the following types of constraints are considered:

Energy demand: $$\sum_i \xi_{i,t} R'_{i,t} \geq d_t, \forall t \quad (8)$$

Turbine capacity: $$\xi_{i,t} R'_{i,t} \leq \overline{P_i}, \forall i, \forall t \quad (9)$$

Flow continuity: $$S_{i,t} = S_{i,t-1} + I_{i,t} + \lambda \sum_{i \in IN} (R'_{i,t} + R''_{i,t}) - \lambda (R'_{i,t} + R''_{i,t}) \quad (10)$$

Monthly maximum storage variation:

$$S_{i,t-1} - S_{i,t} \leq \delta \cdot (S_{i,t}^{\max} - S_{i,t}^{\min}), 0 \leq \delta \leq 1, \forall i, \forall t \quad (11)$$

Minimum and maximum storages: $$S_i^{\min} \leq S_{i,t} \leq S_i^{\max}, \forall i, \forall t \quad (12)$$

Minimum and maximum power releases:

$$R_i'^{\min} \leq R'_{i,t} \leq R_i'^{\max}, \forall i, \forall t \quad (13)$$

Bounds on nonpower release: $$0 \leq R''_{i,t} \leq \infty, \forall i, \forall t \quad (14)$$

where $I_{i,t}$=natural inflow into the ith reservoir during time period t; $S_{i,t-1}$=beginning storage at the ith reservoir; d_t=energy demand during time period t (avgMW); $\overline{P_i}$=effective turbine capacity (MW) for ith power plant; λ=conversion factor from m^3/s to Mm^3; and IN=inflow index from upstream reservoir releases.

The term "avgMW" represents the average MW over a certain period of time. This term is commonly used in Brazil, and all other symbols have been defined. In the continuity equation, it is assumed that evaporation loss from the reservoir is balanced with precipitation onto the reservoir.

The decision variables in the NLP model are $S_{i,t}$, $R'_{i,t}$, and $R''_{i,t}$. The model is nonlinearly constrained with a nonlinear objective function, because the energy production function, $\xi_{i,t}$ is a nonlinear function of storage as well as power and nonpower releases, which can be expressed as follows:

$$\xi_{i,t} = \epsilon_i (HF_{i,t} - HT_{i,t}) \quad (15)$$

$$HF_{i,t} = a_{0i} + a_{1i} S_{i,t} + a_{2i} S_{i,t}^2 + a_{3i} S_{i,t}^3 + a_{4i} S_{i,t}^4 \quad (16)$$

$$HT_{i,t} = b_{0i} + b_{1i} q_{i,t} + b_{2i} q_{i,t}^2 + b_{3i} q_{i,t}^3 + b_{4i} q_{i,t}^4 \quad (17)$$

where $HF_{i,t}$=reservoir forebay water level (m); $HT_{i,t}$=reservoir tailrace water level (m); $q_{i,t}$=total outflow (m^3/s), including power and nonpower releases $(R'_{i,t} + R''_{i,t})$; and ϵ_i=specific productibility (MW/m^3/s/m).

In Eqs. (16) and (17), the forebay and tailrace water levels are expressed by a fourth-order polynomial in terms of storage and total outflow.

The NLP model is most accurate, because it involves no approximation and uses the physically based nonlinear energy production function. Hence, the optimized policy and energy production is most reliable. All gradient-based nonlinear algorithms require an initial policy, i.e., an initial estimate of the solution. Because the optimization problem is also nonconvex, a good initial policy increases the likelihood of reaching the global optimum. A LP model is developed to provide an initial policy for the SLP and NLP models. Additionally, the LP and SLP models are computationally highly efficient and can be utilized to perform sensitivity analysis in connection with capacity expansion studies.

Linear Programming (LP) Model

Our first step is to linearize the NLP model by using a fixed value for the nonlinear energy production function for each hydropower plant. Here we choose the average value of the energy production function, which is obtained from the long-term operational records. The second step is to use a L_1-norm to replace the

L_2-norm in the objectives and then transform them into equivalent linear constraints. Consequently, the LP model is formulated as follows, for Eq. (2*a*), the L_1-norm is

$$\min f_2 = \sum_t \sum_i |S_{i,t} - T_{i,t}| \tag{18}$$

which is equivalent to the following linear programming problem

$$\min_{a_{i,t} \geqslant 0} f_2 = \sum_t \sum_i \alpha_{i,t} \tag{19}$$

subject to

$$-\alpha_{i,t} \leqslant S_{i,t} - T_{i,t} \leqslant \alpha_{i,t} \tag{20}$$

where $\alpha_{i,t}$ is an intermediate variable introduced in the LP formulation. For Eq. (2*b*), the L_1-norm is

$$\min f_2 = \sum_t \left| \sum_i S_{i,t} - T_t \right| \tag{21}$$

which is equivalent to

$$\min_{\beta_t \geqslant 0} f_2 = \sum_t \beta_t \tag{22}$$

subject to

$$-\beta_t \leqslant \sum_i S_{i,t} - T_t \leqslant \beta_t \tag{23}$$

where β_t is an intermediate variable.

Either Eq. (19) subject to Eq. (20) or Eq. (22) subject to Eq. (23) can be used to represent the L_1-norm of objective 2.

The energy complementation in Eq. (5) becomes

$$\min f_5 = \sum_t \left| \sum_i \xi_{i,t} R'_{i,t} - D_T \right| \tag{24}$$

which is equivalent to

$$\min_{\gamma_t \geqslant 0} f_5 = \sum_t \gamma_t \tag{25}$$

subject to

$$-\gamma_t \leqslant \sum_i \xi_{i,t} R'_{i,t} - D_T \leqslant \gamma_t \tag{26}$$

where γ_t is an intermediate variable. A key utilization of the LP model is that its solution provides a good initial policy for the SLP and the NLP models.

Successive Linear Programming (SLP) Model

We now develop a SLP algorithm. The basic idea is to solve the original nonlinear problem via a sequence of localized linear programs so that the final solution is close to that of the NLP model solution. The purposes of developing an SLP model are twofold. First, SLP saves computation time and storage requirements over an NLP approach. Second, the resulting model can be readily solved by a standard LP code. The advantages of LP for reservoir management and operations are well documented (Yeh 1985). This type of approach has been studied by Yeh et al. (1979), Palacios-Gomez et al. (1982), Martin (1983), Grygier and Stedinger (1985), Reznicek and Simonovic (1990, 1992), Tao and Lennox (1991), and Ko et al. (1992). Using a first-order Taylor series expansion, each decision variable is perturbed by a small increment about its current solution

$$\begin{aligned} S_{i,t}^{(k+1)} &= S_{i,t}^{(k)} + dS_{i,t}^{(k)} \\ R_{i,t}'^{(k+1)} &= R_{i,t}'^{(k)} + dR_{i,t}'^{(k)} \\ R_{i,t}''^{(k+1)} &= R_{i,t}''^{(k)} + dR_{i,t}''^{(k)} \end{aligned} \tag{27}$$

We let a vector $\mathbf{x}^{(k)} = [S_{i,t}^{(k)}, R_{i,t}'^{(k)}, R_{i,t}''^{(k)}]^T$ be the current solution (superscript T is the transpose operator) and vector $\boldsymbol{\theta}^{(k)} = [dS_{i,t}^{(k)}, dR_{i,t}'^{(k)}, dR_{i,t}''^{(k)}]^T$ be the unknown increment that is to be determined. Eq. (27) can be expressed as

$$x^{(k+1)} = \mathbf{x}^{(k)} + \boldsymbol{\theta}^{(k)} \tag{28}$$

This linearization is accurate and convergent only if $\boldsymbol{\theta}^{(k)}$ is within a small vicinity of $\mathbf{x}^{(k)}$; therefore, it is necessary to impose additional bounds on $\boldsymbol{\theta}^{(k)}$ as follows:

$$-s \leqslant \boldsymbol{\theta}^{(k)} \leqslant s \tag{29}$$

Vargas et al. (1993) studied the convergence behavior of the SLP method. If s is too small, optimization may encounter infeasibility or a slow rate convergence. However, if s is too large, the method may not converge. In any case, the original bounds, $x_{\min}$ and $x_{\max}$, as represented by Eqs. (12)–(14), should not be violated by the increment. Hence, we have the following new bounds on $\boldsymbol{\theta}^{(k)}$:

$$\max\{x_{\min} - \mathbf{x}^{(k)}, -s\} \leqslant \theta^{(k)} \leqslant \min\{x_{\max} - \mathbf{x}^{(k)}, s\} \tag{30}$$

After linearizing the entire NLP model, Eq. (1) through (11), the SLP model can be formulated as follows:

$$\begin{aligned} \min Z = [&w_1 \nabla f_1^{(k)} + w_2 \nabla f_2^{(k)} - w_3 \nabla f_3^{(k)} + w_4 \nabla f_4^{(k)} + w_5 \nabla f_5^{(k)} \\ &- w_6 \nabla f_6^{(k)}]^T \boldsymbol{\theta}^{(k)} + Z_0^{(k)} \end{aligned} \tag{31}$$

subject to

$$\text{Energy demand:} \quad \sum_i \nabla(\xi_{i,t}^{(k)} R_{i,t}'^{(k)})^T \boldsymbol{\theta}^{(k)} \geqslant d_t - \sum_i \xi_{i,t}^{(k)} R_{i,t}'^{(k)} \tag{32}$$

$$\text{Turbine capacity:} \quad \nabla(\xi_{i,t}^{(k)} R_{i,t}'^{(k)})^T \boldsymbol{\theta}^{(k)} \leqslant \overline{P}_i - \xi_{i,t}^{(k)} R_{i,t}'^{(k)} \tag{33}$$

$$\begin{aligned} \text{Flow continuity:} \quad dS_{i,t}^{(k)} = dS_{i,t-1}^{(k)} &+ \lambda \sum_{i \in IN} (dR_{i,t}'^{(k)} + dR_{i,t}''^{(k)}) \\ &- \lambda (dR_{i,t}'^{(k)} + dR_{i,t}''^{(k)}) \end{aligned} \tag{34}$$

Monthly maximum storage variation:

$$dS_{i,t-1}^{(k)} - dS_{i,t}^{(k)} \leqslant \delta \cdot (S_{i,t}^{\max} - S_{i,t}^{\min}) - S_{i,t-1}^{(k)} + S_{i,t}^{(k)} \tag{35}$$

where

$$Z_0^{(k)} = w_1 f_1^{(k)} + w_2 f_2^{(k)} - w_3 f_3^{(k)} + w_4 f_4^{(k)} + w_5 f_5^{(k)} - w_6 f_6^{(k)} \tag{36}$$

$$\nabla(\bullet) = \left[\frac{\partial \bullet}{\partial S_{i,t}}, \frac{\partial \bullet}{\partial R'_{i,t}}, \frac{\partial \bullet}{\partial R''_{i,t}} \right]^T \quad \text{the gradient operator} \tag{37}$$

The SLP stops if the following convergence criterion is met.

$$\frac{|Z^{(k+1)} - Z^{(k)}|}{Z^{(k)}} \leqslant \boldsymbol{\epsilon} \tag{38}$$

Care must be taken in the selection of s in Eq. (29). If s is too small, infeasibility may result. However, if s is too large, the SLP model may not converge. Fig. 2 shows the SLP flowchart.

Fig. 3 shows the relationship among the LP, SLP, and NLP models. The LP model replaces the nonlinear energy production function by a fixed value for each power plant. This value corre-

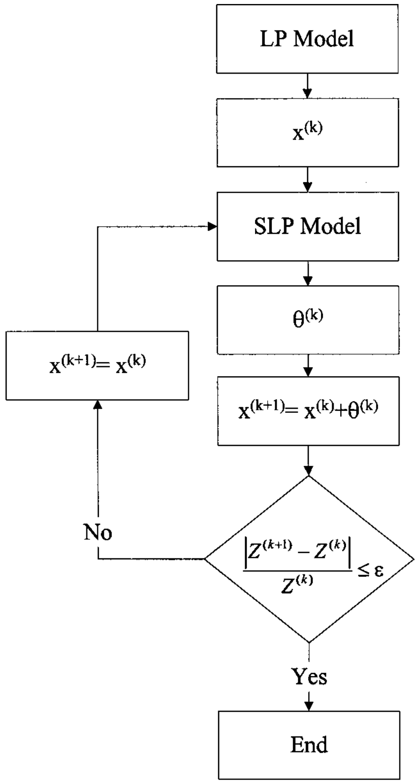

Fig. 2. Flowchart of successive linear programming (SLP)

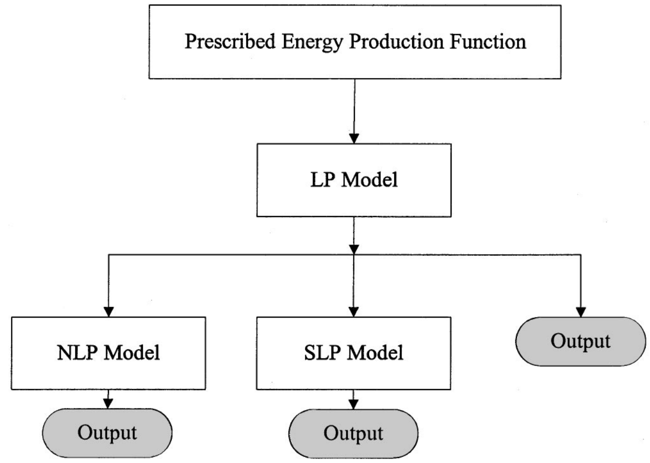

Fig. 3. Relationship among the linear programming (LP), successive linear programming (SLP), and nonlinear programming (NLP) models

sponds to the historical long-term average. Outputs from the LP model include the monthly values of storage, power release and nonpower release at each power plant. These values are used as the initial policy for the SLP and NLP models. To solve the developed LP, SLP, and NLP models, we have explored several linear and nonlinear solvers. MINOS (Murtagh and Saunders 1995) is employed to solve the basic NLP model. With regard to the linear models, we have investigated the following three different solvers: (1) EMNET (McBride, 1985), an algorithm designed for solving embedded generalized network flow problems; (2) the MINOS LP, which is based on the standard simplex method of linear programming; and a primal-dual interior-point algorithm PCx (Czyzyk et al. 1999), which uses a logarithmic barrier function to accommodate constraints and uses Newton's method to solve the Karush-Kuhn-Tucker (KKT) equations. In many hydropower and hydrothermal optimization problems, it has been shown that the interior point method is much faster than the MINOS LP for large-scale optimization (Momoh et al. 1999b). Medina et al. (1998) made a comparative evaluation of several interior point codes. Ponnambalam et al. (1989) applied the Karmarkars interior point LP algorithm to optimize the operations of a multireservoir system. Christoforidis et al. (1996) showed that very large midterm or long-term resources scheduling problems could be successfully solved using the interior point method.

Our model uses Excel and Visual Basic in conjunction with FORTRAN. All input data for the objectives [Eqs. (1)–(6)], the constraints [Eqs. (8)–(14)], and the energy production functions [Eqs. (15)–(17)] are listed in the Excel worksheets. Excel creates a convenient user-interface for operating the hydropower model.

Table 1. Linear Programming (LP) Scenarios and Results, Case 1

Scenarios	1	2	3	4	5
Number of planning years	5	5	5	10	15
System version	Paranapanema	Parana	Entire	Entire	Entire
Number of nodes	8	39	75	75	75
Storage variation (%)	100	100	10	10	10
Number of variables	1,620	7,200	13,680	27,360	41,040
Number of constraints	600	2,460	9,180	18,360	27,540
(a) Objective value (10^9)					
MINOS	0.081086	1.784977	1.401181	2.490497	6.693066
EMNET	0.081086	1.784977	—	—	—
PCx	0.081086	1.784977	1.401181	2.490497	6.693069
(b) CPU (s)					
MINOS	1.53	56.74	246.50	1,238.46	2,912.09
EMNET	1.70	133.41	—	—	—
PCx	0.28	1.76	8.13	24.83	41.30
Ratio (MINOS/PCx)	5.46	32.24	30.32	59.96	70.51
Ratio (EMNET/PCx)	6.07	75.80	—	—	—

Table 2. Model Dimension and Simulation Scenario for the Entire System, Case 2

Scenario	Dimension
Simulation period	1991–1995
Installed capacity	69,375 MW
Number of nodes	75
Storage variation	100%
Number of variable	13,680
Number of constraints	9,180

Three optimization models—LP, SLP, and NLP—are programmed using the FORTRAN language and compiled as a dynamic-link library (DLL) for Excel. The optimization solvers (MINOS, EMNET, and PCx) are linked with Fortran codes. After the optimum results are obtained from the Fortran codes, these results are stored in the Excel worksheets and visualized with monthly energy production graphs, monthly storage variation graphs, and so forth. Visual Basic manipulates the input and output data between Excel and FORTRAN. The present version runs on a PC with a 500 MHz Pentium III processor and 160 MB RAM. In the case study section, we will discuss the computational efficiency of various solvers.

Case Study and Results

Case 1: Comparison of LP Solvers

The first case study tests the validity and computation efficiency among the MINOS LP, EMNET, and PCx. Three system configurations are considered ranging from a small subsystem to the entire system (Fig. 1): (1) Paranapanema subsystem (nodes 32–39); (2) Parana subsystem (Nodes 1–40, not including Node 23); and (3) the entire system (Nodes 1–75). The objective functions considered are the minimization of total stored potential energy (f_1) and individual storage deviations from targets (f_2). We use historical inflow data for all scenarios.

The system sizes and optimization results of the five scenarios analyzed are shown in Table 1. The optimized objective values obtained from the MINOS LP, EMNET, and PCx are virtually identical. This shows that the LP solvers are valid. The first two scenarios show that PCx and the MINOS LP are more efficient than EMNET for small-scale problems. For large-scale problems, PCx outperforms the MINOS LP, and PCx is almost 71 times faster than the MINOS LP for Scenario 5.

Table 3. Optimized Total Energy Production and Computation Time from Linear Programming (LP), Successive Linear Programming (SLP), and Nonlinear Programming (NLP) Models, Case 2

Model	Solver	Objective function f_3 (avgMW)	CPU time
LP[a]	PCx	49,698	17 (s)
SLP[b]	PCx	49,965	3.73 (min)
NLP[c]	MINOS	49,974	2.19 (h)

[a]Linear programming.

[b]Successive linear programming.

[c]Nonlinear programming.

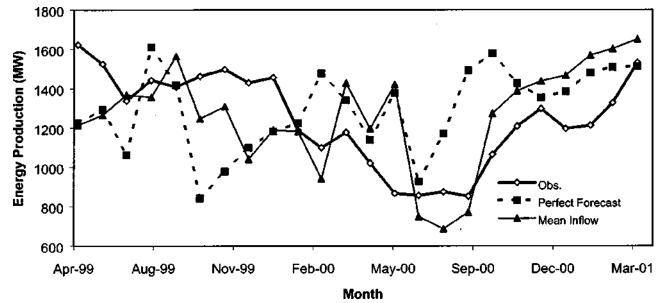

Fig. 4. Comparison against historical monthly energy production by maximizing f_3, Paranapanema subsystem, Case 3

Table 4. Paranapanema Subsystem and Simulation Scenario, Case 3

Scenario	Dimension
Simulation period	April 1999 to March 2001
Installed capacity	2,308 (MW)
Number of nodes	9
Storage variation	100%
Number of variables	648
Number of constraints	240

Table 5. Comparison of Total Energy Production by Maximizing f_3, Paranapanema Subsystem, Case 3

Model	Objective function, f_3 (avgMW)	Increase (%)
Mean-inflow forecast	1,264	4.09
Perfect forecast	1,297	6.86
Historical	1,214	—

Case 2: Model Application to Large-scale System

The developed LP, SLP, and NLP models are applied to the Brazilian hydropower system with 75 hydropower plants, schematically represented by the 75 nodes in Fig. 1. From the testing results of the three LP solvers mentioned earlier, we have found that in all cases PCx is much faster than the MINOS LP and EMNET. Because the constraint matrix is not dominated by network substructure, EMNET turned out to be ineffective in this particular application. Hence, we have adopted PCx as the LP solver for the rest of the studies.

Table 2 lists the model dimension and simulation scenario. The simulation period is from 1991 to 1995. Monthly time period and historical inflows are used in the simulation. The corresponding optimization model has 13,680 variables and 9,180 constraints. In this example, we chose Objective 3 (f_3) for demonstration. Table 3 shows the optimization results and computation time. As can be seen, there is no major difference in terms of the maximized total energy production among the three models.

One important characteristic of the basic model is that it can be used to analyze system operations by two different approaches—linear and nonlinear. The LP model replaces the energy production functions by their corresponding average values. The SLP model uses a Taylor series expansion about the initial policy provided by the LP model and achieves convergence to a local optimum through successive iterations. The objective (f_3) was chosen to maximize the total energy production. Reservoir storage was constrained by flood control reservation during the

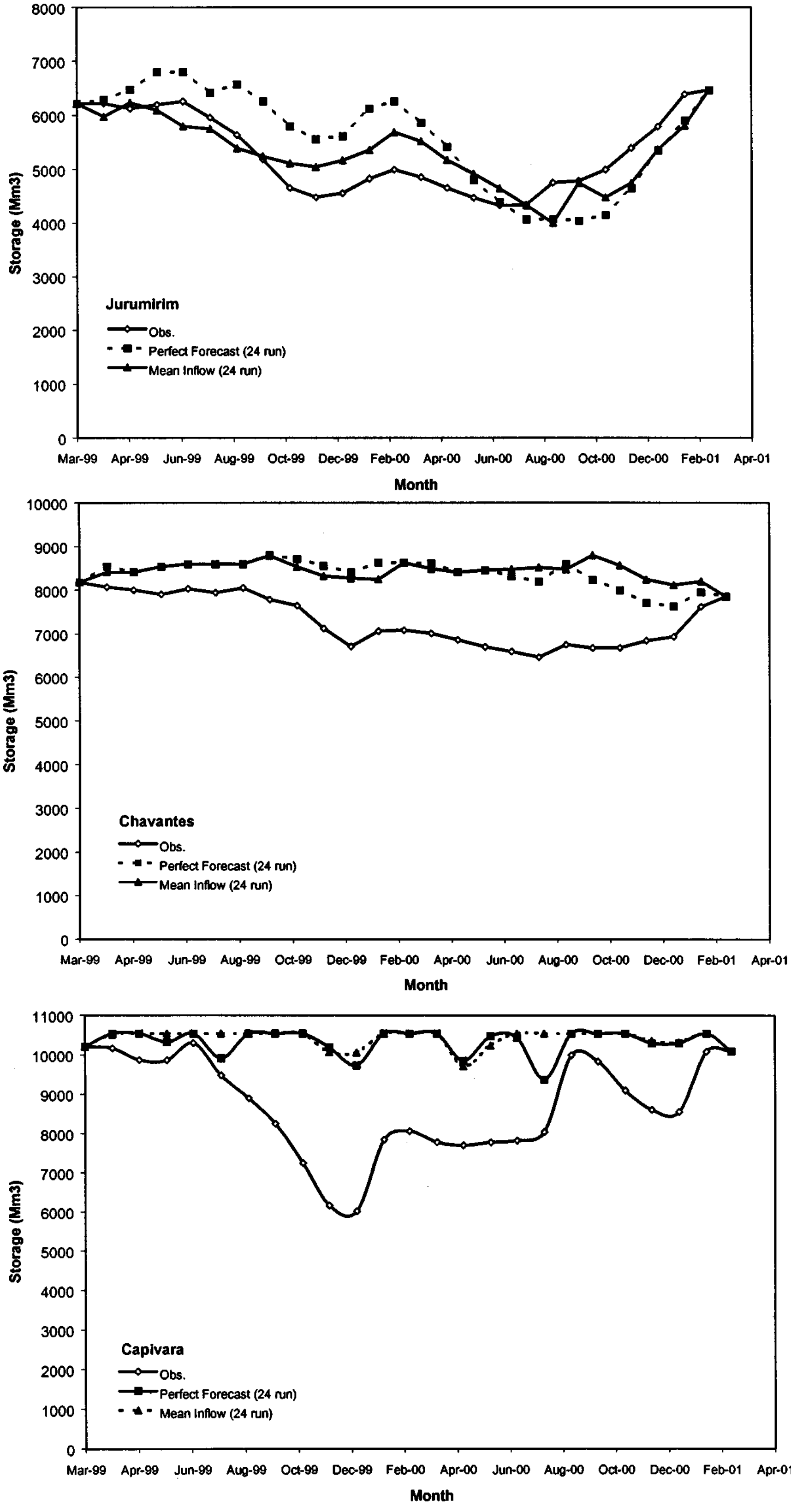

Fig. 5. Comparison against historical monthly storage variations at the three storage reservoirs, Paranapanema Subsystem, by maximizing the total energy production (f_3), Case 3

Table 6. Comparison of Total Energy Production by Minimizing f_5, Paranapanema Subsystem, Case 3

Model	Objective function, f_5 (avg MW)	Increase (%)
Mean-inflow forecast	1,267	4.33
Perfect forecast	1,296	6.72
Historical	1,214	—

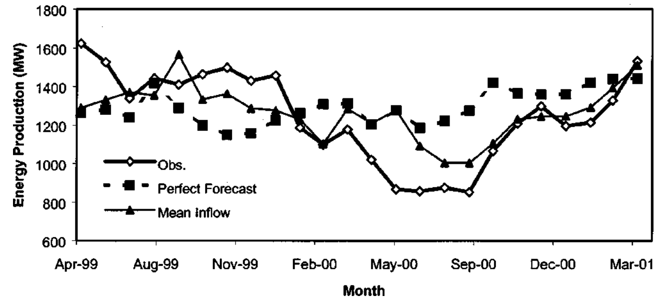

Fig. 6. Comparison against historical monthly energy production, Paranapanema subsystem, by minimizing the energy complementation (f_5), Case 3

wet season, and no constraint was imposed on the maximum allowable monthly storage variation.

The results obtained show that the LP model produced 0.6% less total energy and took only 17 s of CPU time to solve (Table 3). The total energy produced by the SLP and NLP models is virtually the same (Table 3). A close examination of the results (not shown here) indicates that the storage varies more frequently in the NLP model than in the LP model. In general, the LP model produced smoother storage transitions. However, with a good initial policy provided by the LP model, the SLP model converged and produced excellent results for this particular example. The success of the SLP model is problem dependent. For a convex problem, it can be shown that the final solution of the SLP converges to the solution of the original nonlinear problem. For problems that are nonconvex, which is the case here, the final solution of the SLP is not guaranteed to be close to that of the original nonlinear solution.

Overall, when compared with the NLP model, the LP model produced good results in terms of the total energy production, recognizing that there are differences in reservoir storage variations, but these factors are not important for planning purposes. Therefore, we conclude that the LP model can be used as an efficient tool to explore various design parameters in connection with feasibility studies. With a good initial policy provided by the LP model, the SLP model produced excellent results and cut down the CPU time from hours to minutes. Of course, the NLP model is the most accurate one among the three; therefore, we proceed to test it against the historical operational records.

Case 3: Comparison with Historical Records

The ONS (Operador Nacional do Sistema) is in charge of the Brazilian hydropower system operation. To provide guidelines for real-time operations, ONS first uses an energy strategy model, NEWAVE, which is an operation-planning model and generates operational strategy for the next 60 months. This model aggregates all reservoirs into a single reservoir and is solved by stochastic DP. The solution from the stochastic DP is then disaggregated by simulation, i.e., the NEWAVE operational strategy is used as the input for a simulation model to define the energy production for each reservoir on a monthly basis. The weekly, daily, and hourly operations are obtained by simulation with flow and demand forecasting models. The hourly operations are used for real-time operations (Barros 2000). The historical monthly energy production is obtained by adding the hourly energy production for the particular month under consideration.

The Paranapanema subsystem is located in the Paranapanema River, a tributary of the Parana River. It consists of three storage power plants and five run-of-river power plants with an installed capacity of 2,308 MW. The monthly historical energy production from April 1999 to March 2001 is shown in Fig. 4. To optimize the hydropower production for this period, the model has 648 decision variables and 240 constraints. The storage variation is unrestricted, i.e., $\delta = 1$ (Table 4). First, we maximize the total energy production (f_3), subject to flood control and all other constraints.

In real-time operation, reservoir operators rely upon inflow (natural inflow) forecasts with frequent updating. It is well understood that inflow forecast plays an important role in real-time operation. To compare the NLP results against the historical operational records, we use the following two types of inflow forecast models as inputs to the optimization model: (1) perfect forecast; and (2) mean-inflow forecast. The perfect forecast model simply uses the observed historical inflows as the forecasts. This implies that there is no forecast error. In real-time operation, of course, this is not realistic, but the results obtained from the optimization model based on the perfect forecast model will provide an upper bound for the energy production for this particular period under consideration. The mean-inflow forecast model uses the monthly mean as its forecast. Because most of the streamflow forecast models should be able to provide inflow forecasts that are better than simply using the mean values, the results obtained from the optimization model will provide an estimate of the energy production that might be achievable in real-time operation. To simulate real-time operation, the state of the system is updated on a monthly basis whenever new information becomes available at the beginning of each month, such as the previous month's observed inflow.

Table 5 shows the comparison against the historical operational records. It can be seen that during the same period of operation with the same initial and ending storages, the NLP model produced 6.86% more energy using the perfect forecast model and 4.04% more energy using the mean-inflow forecast model. As we have mentioned, a small percentage increase in total energy production usually translates into large benefits. A 4.04% increase in power production is in reality quite substantial. Fig. 4 shows a comparison of the monthly energy production against the historical operational records. The optimization model takes advantage of the inflow forecast by minimizing spill and maximizing head within the feasible region of the decision space. Hence, the energy production distribution obtained by the optimization model is expected to be different from the historical operational records, which did not use optimization.

Fig. 5 presents a comparison of the monthly storage variations of the three large storage reservoirs against the historical operational records. The total storage increased approximately 15% in the Chavantes reservoir (Node 33) and approximately 19% in the Capivara reservoir (Node 37) using either the perfect forecast or the mean-inflow forecast model. It can be seen that the NLP model is able to take advantage of the inflow forecast by keeping

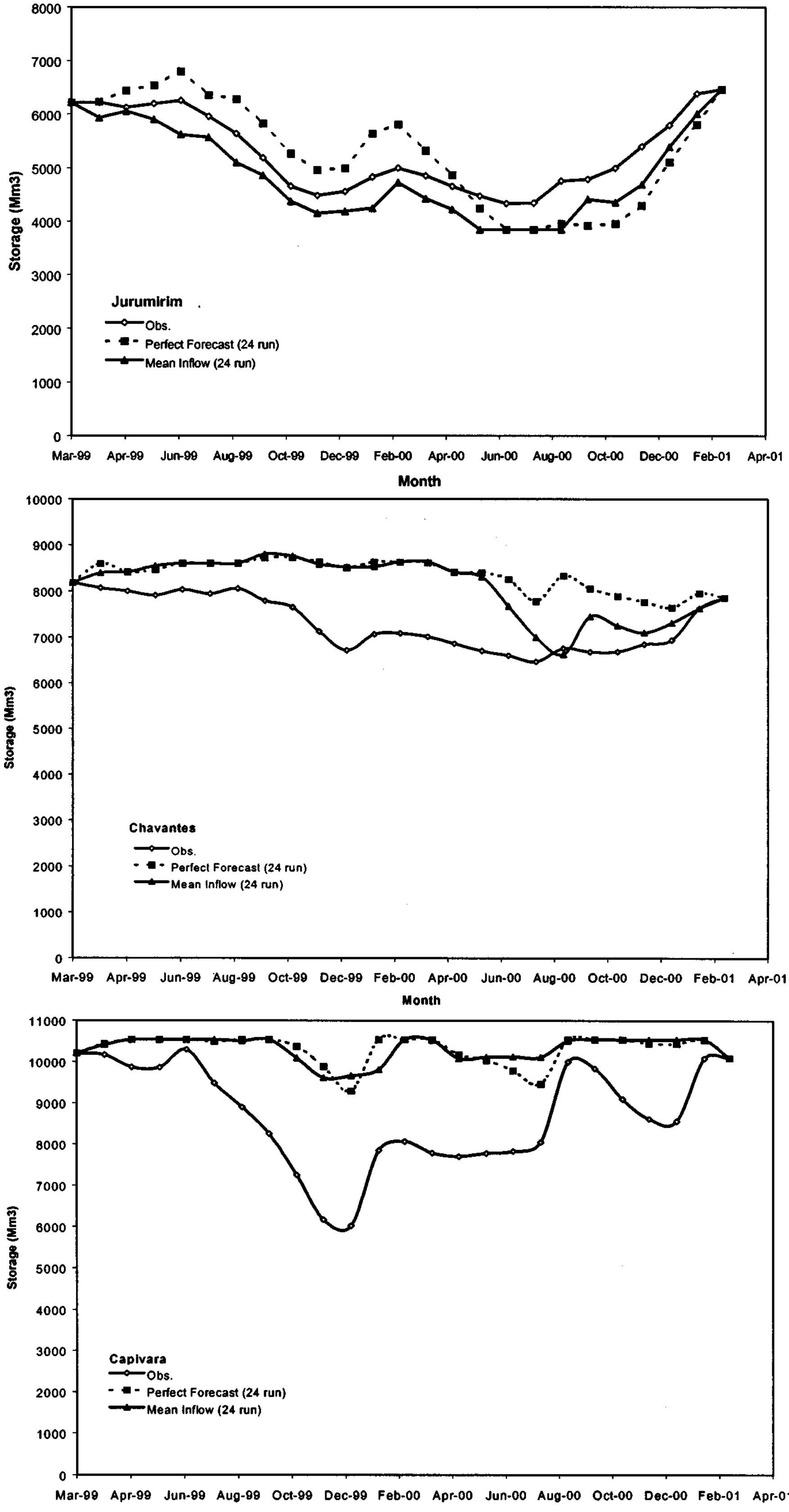

Fig. 7. Comparison against historical monthly storage variations at the three storage reservoirs, Paranapanema subsystem, by minimizing the energy complementation (f_5), Case 3

the storage, and thus the head, at a higher level to maximize energy production and to minimize spill.

We have also compared the performance of the NLP model against the historical operational records using Objective 5 (f_5), i.e., minimizing energy complementation. Table 6 summarizes the results. Again, the NLP model produced more energy using either the perfect inflow or the mean-inflow forecast model. For a given total energy demand, D_T, we note that both Objectives 3 and 5 seek to maximize the hydropower energy production, but in a different way. Objective 5 is a quadratic function in terms of energy production; thus, the optimization model will maximize the energy production, but, at the same time, minimize the variation of energy production distribution over the planning horizon. Fig. 6 shows such characteristics in the optimized monthly energy production when compared to the historical operational records. The optimized energy production distribution using the perfect forecast model is especially uniform. Fig. 7 compares the monthly storage variations at the three storage reservoirs. Again, the total storage increased more than 11% in the Chavantes reservoir and about 18% in the Capivara reservoir using either inflow forecast model.

Summary and Conclusions

An NLP model has been formulated and applied to the Brazilian hydropower system, one of the largest in the world with an installed capacity of 69,375 MW. The model was solved by NLP. Additionally, the NLP model was linearized and solved by LP and SLP. The LP model uses the L_1-norm and replaces the nonlinear energy production functions by their corresponding average values. The SLP model uses a Taylor series expansion about an initial policy provided by the LP model, and convergence to a local optimum is achieved via a sequence of localized linear programs. The LP, SLP, and NLP models were designed to perform different tasks with different objective functions. The multiobjective optimization problem can be solved by the weighting method or solved as a single objective optimization problem by treating other objectives as constraints.

The impact of nonlinearity was analyzed, and the results show that, for planning purposes, the LP model is sufficient. The proposed SLP produced excellent results with fast convergence. It can also be used for planning purposes. But the NLP model is the most accurate and particularly suitable for real-time operation. The performance of the NLP model was compared against the historical operational records. To simulate real-time operation, the NLP model takes advantage of the inflow forecast with frequent updating. We have used two inflow forecast models to estimate the upper and lower bounds in the NLP model performance. The results are extremely promising in that the NLP model meets the demand and produces more energy by maximizing storage (thus, the head) and by minimizing spill. The NLP model is extreme useful for setting up guidelines for real-time operation.

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Notation

The following symbols are used in this paper:

c_i' = weighting coefficient for power release;
c_i'' = weighting coefficient for nonpower release (spill);
D_C = contractual demand (system firm energy) during time period t (avgMW);
D_T = total energy demand, including hydropower, thermal energy, and nuclear energy;
d_t = energy demand during time period t (avgMW);
$HF_{i,t}$ = reservoir forebay water level (m);
$HT_{i,t}$ = reservoir tailrace water level (m);
$I_{i,t}$ = natural inflow into the ith reservoir during time period t;
$\overline{P}_i$ = turbine capacity (MW) for ith power plant;
p_t = energy price during time period t;
$q_{i,t}$ = total outflow (m^3/s), including power and non-power releases ($R_{i,t}'+R_{i,t}''$);
$R_{i,t}'$ = power release from reservoir i during time period t (m^3/s);
$R_{i,t}''$ = non-power release from reservoir i during time period t (m^3/s);
$S_{i,t}$ = ending storage in the ith reservoir at the end of time period t;
$S_{i,t-1}$ = beginning storage at the ith reservoir;
$T_{i,t}$ = specified ending storage target at the ith reservoir at the end of time period t;
T_t = specified ending storage target for the summation of the ending storage vector of the entire system at the end of time period t;
ϵ_i = specific productibility (MW/m^3/s/m);
λ = conversion factor from m^3/s to Mm^3; and
$\xi_{i,t}$ = energy production function in MW/m^3/s/month.

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